ENGINEER

international scientific journal

ISSUE 3, 2025 Vol. 3

E-ISSN
3030-3893

ISSN
3060-5172

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ENGINEER

A bridge between science and innovation

E-ISSN: 3030-3893

ISSN: 3060-5172

VOLUME 3, ISSUE 3

SEPTEMBER, 2025

engineer.tstu.uz

TASHKENT STATE TRANSPORT UNIVERSITY

ENGINEER

INTERNATIONAL SCIENTIFIC JOURNAL
VOLUME 3, ISSUE 3 SEPTEMBER, 2025

Founder of the international scientific journal “Engineer” – Tashkent State Transport University, 100167, Republic of Uzbekistan, Tashkent, Temiryo‘lchilar str., 1, office: 465, e-mail: publication@tstu.uz.

The “Engineer” publishes the most significant results of scientific and applied research carried out in universities of transport profile, as well as other higher educational institutions, research institutes, and centers of the Republic of Uzbekistan and foreign countries.

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Tashkent State Transport University had the opportunity to publish the international scientific journal “Engineer” based on the **Certificate No. 1183** of the Information and Mass Communications Agency under the Administration of the President of the Republic of Uzbekistan. **E-ISSN: 3030-3893, ISSN: 3060-5172.** Articles in the journal are published in English language.

Numerical modeling of two-phase filtration processes in interconnected reservoir layers of oil fields

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Abstract: In this article, dedicated to the mathematical modeling of two-phase (oil-water and oil-gas) filtration processes in oil and gas fields, the development of numerical algorithms, and the creation of software tools. The study presents a mathematical model of the filtration process through dynamically interconnected layers in a porous medium. Based on in-depth analysis, numerical solutions were proposed for solving the two-dimensional boundary value problem using finite difference methods and iterative computation algorithms. The developed software complex enables real-time calculation of oil pressure, saturation levels, and key hydrodynamic indicators, which are presented through 3D visualizations.

The results obtained from the study offer opportunities for efficient design, analysis, and management of oil field development. This software tool has both theoretical and practical significance, providing high accuracy and efficiency in modeling and forecasting filtration processes. The presented work opens new directions in science, engineering, and practice, and contributes to the improvement of oil field development strategies.

Keywords: filtration process, mathematical modeling, numerical algorithm, porous media

1. Introduction

In recent years, significant attention has been given globally to the development, enhancement, and advancement of mathematical models of gas-hydrodynamic processes in porous media. There is a growing focus on the application of numerical methods and modern computer technologies to solve linear and nonlinear boundary value problems of filtration. One of the main objectives in this field remains the creation of automated systems based on the development of software for determining and forecasting the main indicators of oil and gas field operations, as well as the study of unsteady filtration processes using modern information technologies.

In developed countries such as the USA, France, China, the UAE, Iran, Russia, Kazakhstan, Azerbaijan, and others, extensive practical work is being carried out to develop mathematical models, computational algorithms, and software for simulating unsteady filtration processes of oil and gas. In some countries, particularly Russia and Kazakhstan, scientific research is being conducted to model multiphase (oil-water, oil-gas, and oil-water-gas systems) filtration processes in porous media, calculate key performance indicators of oil and gas fields, develop software systems, construct 3D models of geological and hydrodynamic objects, perform computational experiments to study filtration processes, and analyze the obtained results.

One of the key tasks in this field is to study the complex movement of oil and gas in multilayer porous media and to build mathematical models that accurately reflect real-world objects. Developing computational algorithms and creating automated systems are also among the primary objectives. Furthermore, one of the critical research directions is the scientific justification and development of numerical methods and efficient algorithms for solving nonlinear problems in filtration domains with complex configurations.

Worldwide, scientific research continues in the development of mathematical models for two-phase filtration processes, particularly oil-gas systems in porous media, the creation of algorithms for calculating the key performance indicators of oil and gas fields, computer modeling systems, and the construction of 2D and 3D geological and hydrodynamic models. Computational experiments are carried out to analyze the filtration processes, and the results are used for visual and analytical investigations. Specifically, studying complex fluid dynamics in single and dynamically interconnected multilayer porous media, constructing accurate mathematical models for real-world conditions, designing computational algorithms, and developing automated systems remain among the most critical objectives in this field.

Mathematical models of multiphase flows of liquids and gases in porous media are based on the general laws of continuum mechanics and are reduced to systems of nonlinear partial differential equations with corresponding initial, boundary, and internal conditions. In general, these systems do not have analytical solutions. Therefore, to build mathematical models of multiphase filtration flows, various simplifications are employed that allow for analytical solutions. However, analysis of real oil and gas field conditions shows that the solutions of simplified mathematical models often do not correspond to the actual parameters of the reservoir. As a result, the estimates derived from these models differ from real values. In this regard, it is advisable to develop general mathematical models, algorithms, and software tools that are suitable for analyzing and forecasting real-world oil and gas fields.

2. Research methodology

Modern methods of numerical simulation effectively implemented on contemporary computers have become new

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operational tools for scientific research. In this case, numerical simulation serves not only as a method for obtaining quantitative characteristics but also as a means of establishing the governing laws of the processes under study. Thus, based on a physical model that encompasses the main aspects of the process, an appropriate mathematical model in the form of a system of equations solvable by numerical methods on a personal computer can be developed.

Currently, there are mathematical models that describe the joint filtration processes of fluids in porous media. The development of these models has greatly benefited from the contributions of scientists such as N.N. Veregin, V.N. Nikolayevsky, V.M. Shestakov, E.S. Zakirov, B.B. Lopukh, F.B. Abutaliev, D.F. Fayzullaev, R. Sadullayev, and others, including:

N.N. Veregin is known for his fundamental research in the mathematical modeling of filtration processes in porous media. His studies focused on nonlinear systems of equations describing multiphase flows, and he developed physical models to calculate pressure and saturation distributions in complex reservoir structures[2].

V.N. Nikolayevsky was a leading expert in subsurface hydromechanics and geophysical modeling. He analyzed fluid flow in porous media based on relative permeability coefficients and demonstrated the effects of reservoir deformation and mechanical conditions on filtration processes[3].

V.M. Shestakov contributed significantly to computational modeling by developing numerical solutions for filtration models in one-, two-, and three-dimensional cases. His work focused on adapting these models to real reservoir conditions using advanced numerical methods[4].

E.S. Zakirov is one of Uzbekistan's leading scholars in the fields of filtration and hydrodynamics. His research analyzed the evolutionary distribution of pressure in reservoirs and emphasized modeling the dependence of fluid properties—such as pressure, density, and temperature—on filtration behavior[5].

B.B. Lopukh offered strong algorithmic approaches for solving filtration problems using mathematical modeling and computer technologies. He developed practical methods for calculating physical parameters in two-phase flows under real reservoir conditions, grounded in experimental data[6,21].

F.B. Abutaliev conducted hydraulic analysis of complex oil and gas pipeline systems. His models addressed pressure drop, flow imbalance, and the movement of gas-liquid mixtures, and provided precise solutions for such systems[7].

D.F. Fayzullaev carried out extensive research on mathematical and numerical modeling of multiphase filtration processes. He proposed analytical solutions to filtration equations and developed simplified models aimed at approaching real-world conditions more closely[8].

R. Sadullayev developed scientific approaches to the mathematical modeling of interactions in reservoir-gas-liquid systems. He created models and algorithms that account for the dynamic changes in physical properties of the reservoir, such as porosity and permeability, during filtration processes[9].

Many authors have studied various aspects of fluid and gas filtration in porous media and the creation of corresponding mathematical models. The origins of studying underground hydromechanics trace back to the work of French engineer A. Darcy (1803–1858), who conducted numerous experiments on water filtration through vertical sand filters during a water supply project in Dijon (France). These experiments laid the groundwork for solving problems in modern hydrodynamics and hydromechanics and analyzing them mathematically [1].

In particular, current research in Uzbekistan and internationally focuses on solving filtration problems of fluids and gases in porous media, obtaining their analytical and approximate solutions, developing mathematical and computer simulations, constructing imitation and simulation models, and applying various new and improved computational methods and algorithms. These efforts, when combined with the latest advancements in technology, help present models that facilitate visualization and understanding of the overall process.

To study and analyze these problems, numerous scientific works by foreign and domestic scholars have been examined, leading to the following findings:

For instance, the works of K. Aziz, E. Settari, and N.B. Lopukh focus on the mathematical and numerical simulation of single and multiphase oil and gas field development processes, as well as on methods for solving one- and multi-dimensional problems based on boundary conditions[6,21].

Currently, a new research methodology—mathematical modeling and computational experimentation—is emerging. This methodology involves replacing the actual object with its "image"—a mathematical model—and further studying it using algorithms implemented on computers. This approach enables fast and cost-effective testing of various characteristics. The methodology of mathematical modeling is rapidly evolving and encompasses new directions, ranging from analyzing physical, economic, and social processes to developing and managing complex technical systems.

Computational experiments enable faster and more efficient research. For example, when analyzing the hydraulic regimes of technological segments of oil pipelines, the proposed computational algorithms consider the technological segment (including the main and intermediate pump stations) as a single hydraulic system. These aspects are discussed in the scientific works of F.B. Abutaliev, M.B. Baklushin, Y.S. Erbekov, U.U. Umarov, and others. Introducing flow modifiers to counteract the uneven movement of gas at any stage affects the operation of other stages. Therefore, calculating the regime of a technological segment, predicting overall oil and gas expenditures, and increasing operational efficiency are possible with algorithms developed by V.V. Yakovlev, Yu.I. Kalugin, N.G. Stepova, and others[22].

Research has also addressed increasing natural gas extraction volumes by redistributing flow rates in production wells based on the mathematical model of filtration and 2D visualization of two-phase multicomponent hydrocarbon mixtures [3].

Other studies have investigated the complex dynamic processes of oil displacement by gas and water under reservoir conditions. Effective mathematical models and numerical algorithms have been developed for three-phase filtration processes of oil, gas, and water in porous media [4]. Some researchers, such as Luis Cueto-Felgueroso, Xiaojing Fu, and Ruben J., have analyzed methods for solving the two-phase flow problem using the Buckley-Leverett model without considering capillary and gravitational forces. They emphasize the importance of accurately determining relative phase permeability functions that depend on saturation coefficients for reliable modeling [10].

In modeling unsteady two-phase systems (oil-gas, oil-water), researchers like B. Kh. Khuzhayorov and V.F. Burnashev have used integral methods to solve nonlinear differential equations and calculate pressure distributions and flow parameters. M.V. Vasilyeva and G.A. Prokopev have studied hydrodynamic parameters of oil-gas-condensate reservoirs using finite difference and iterative convergence methods[11,12].

S.V. Zvonarev has emphasized that mathematical modeling is broadly applied in sciences like mathematics, physics, and biology, and must meet the following requirements: clearly formulated assumptions based on experiments, adequacy analysis of the model, and precision of computational algorithms. Modeling complex systems requires distinguishing between mathematical and non-mathematical concepts and using appropriate mathematical tools[13].

Baxtiy Nikolay Sergeyevich has focused on analyzing multiphase filtration models, including pressure-driven and pressureless flows, and methods for computing pressure over time (e.g., IMPES scheme). He developed analytical and numerical solutions for subsurface aquifer flow and gas release problems within the "TechScheme" model[14]. Other researchers like R.M. Siddikov, D.D. Filippov, and D.A. Mitrushkin have developed computational algorithms and software for simulating unsteady three-phase fluid flow in "Layer-Well-EOR" systems. Studies such as have used finite difference methods for approximating velocity and pressure in heterogeneous porous media and Galerkin methods for discretizing the saturation equation via artificial diffusion[15].

Mathematical modeling of multiphase fluid flows in porous media is of great practical importance for oil and gas production. Accurate numerical modeling of specific hydrodynamic problems requires precise physical-mathematical formulations, knowledge of the parameters and initial data, and confidence in their accuracy. Applied numerical methods must be economical and broadly applicable to various types of problems[7].

In applied mathematics, solving problems on computers follows a technological chain: research object → mathematical model → algorithm (numerical methods) → computer program → computational experiment → analysis (or comparison with experimental data). The objective of mathematical technology lies in the computational component of this chain: resulting in the chain "object → model → algorithm → program → computation". This technology enables the analysis, forecasting, and control of unsteady oil and gas extraction processes under reservoir conditions.

Mathematical model

Filtration can be described in terms of interpenetrating and interacting media in the model of X.A. Rakhmatullin. In this case, the velocity of motion in the layers is assumed to be zero, and the liquid and gas multicomponent media move relative to each other and to the structure. In this case, difficulties arise in describing the interaction force between the constituent phases and the components of a unit volume of the medium. In this regard, within the framework of this chapter, we turn to the nonlinear Darcy law and apply it to two-dimensional filtration in Cartesian coordinates[1].

In a gas layer of variable thickness, we separate the elementary volume $\partial x \partial y h\{x,y\}$. Here $h\{x,y\}$ is the value of the layer thickness at the point with coordinates x and y (Fig.1).

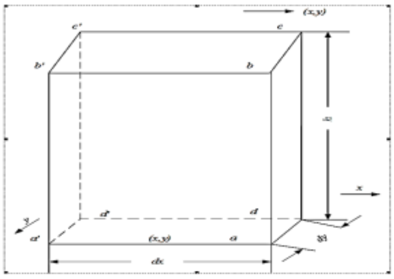

Fig. 1. Volume of elementary layers

In this case, if gravitational forces are neglected, the fluid velocity is described using the following formulas.

Based on these assumptions, the following equations from the theory of filtration are used to mathematically model the unsteady (non-stationary) two-dimensional filtration process of two- and three-phase fluids in a porous medium [12]:

Continuity equation:

$$div\left(\frac{1}{B_o}\vec{v}_o\right) = -\frac{\partial}{\partial t}\left(\frac{1}{B_o} m S_o\right) + q_o. \quad (1)$$

- for the oil phase:

$$div\left(\frac{1}{B_w}\vec{v}_w\right) = -\frac{\partial}{\partial t}\left(\frac{1}{B_w} m S_w\right) + q_w \quad (2)$$

- for the water phase:

$$div\left[\frac{R_s}{B_o}\vec{v}_o + \frac{1}{B_g}\vec{v}_g\right] = -\frac{\partial}{\partial t}\left[m\left(\frac{R_s}{B_o}S_o + \frac{1}{B_g}S_g\right)\right] + q_g + R_s \cdot q_o \quad (3)$$

- for the gas phase:

In addition to the above filtration equations, the following relationships are also used:

$$S_o + S_w + S_g = 1, \quad (4)$$

$$S_o + S_w = 1, \quad (5)$$

$$S_o + S_g = 1, \quad (6)$$

$$P_o - P_w = P_{cow} = f_1(S_w, S_g), \quad (7)$$

$$P_g - P_o = P_{cog} = f_2(S_w, S_g), \quad (8)$$

here, P_{cow} and P_{cog} represent the capillary pressures in the oil-water and oil-gas systems, respectively.

The simultaneous filtration of two or more fluids and gases in a dynamically connected two-layer porous medium is a highly complex problem characterized by the following features: the coefficients of the equations depend on time, spatial coordinates, and fluid saturation; the capillary pressure and relative phase permeabilities in the system (P_{cog}, K_g, K_o) are determined from experimental data as functions of saturation.

The mathematical model under consideration is based on the following assumptions[1]:

- The movement of oil and gas in the porous medium is linear and governed by Darcy's law;
- The permeability coefficients of both layers in the vertical direction are the same;
- The upper and lower layers are dynamically connected through poorly permeable structures;
- The thicknesses of both layers are constant;
- The properties of the fluids do not change over time;
- Gas dissolves in oil;
- The fluids and gases in both layers are at a constant temperature and in thermodynamic equilibrium.

https://doi.org/10.56143/3030-3893-2025-3-17-25 A bridge between science and innovation

Taking these assumptions into account and considering two-phase filtration properties in a one-dimensional setting for a dynamically connected two-layer porous medium, the mathematical model of the problem can be expressed as the following system of differential equations [16]:

$$\begin{cases} \frac{\partial}{\partial x}\left[\frac{K_{1o}}{\mu_o}k_1\rho_{1o}\left(\frac{\partial P_{1o}}{\partial x}\right)\right] = \frac{\partial}{\partial t}\left[m\rho_{1o}(1-S_{1g})\right], \\ \frac{\partial}{\partial x}\left[R_s\frac{K_{1o}}{\mu_o}k_1\rho_{1o}\left(\frac{\partial P_{1o}}{\partial x}\right)\right] + \frac{\partial}{\partial x}\left[\frac{K_{1g}}{\mu_g}k_1\rho_{1g}\left(\frac{\partial P_{1g}}{\partial x}\right)\right] = \\ = \frac{\partial}{\partial t}\left[m\rho_{1o}R_s(1-S_{1g}) + m\rho_{1g}S_{1g}\right] - \frac{\rho_{1g}k_{\Pi}}{h_1h_{\Pi}\mu_g}(P_{2g}-P_{1g}); \end{cases}$$

$$\frac{\partial}{\partial x}\left[\frac{K_{2o}}{\mu_o}k_2\rho_{2o}\left(\frac{\partial P_{2o}}{\partial x}\right)\right] = \frac{\partial}{\partial t}\left[m\rho_{2o}(1-S_{2g})\right];$$

$$\frac{\partial}{\partial x}\left[R_s\frac{K_{2o}}{\mu_o}k_2\rho_{2o}\left(\frac{\partial P_{2o}}{\partial x}\right)\right] + \frac{\partial}{\partial x}\left[\frac{K_{2g}}{\mu_g}k_2\rho_{2g}\left(\frac{\partial P_{2g}}{\partial x}\right)\right] = = \frac{\partial}{\partial t}\left[m\rho_{2o}R_s(1-S_{2g}) + m\rho_{2g}S_{2g}\right] + \frac{\rho_{2g}k_{\Pi}}{h_2h_{\Pi}\mu_g}(P_{2g}-P_{1g}) + Q_2; \quad (9)$$

$$P_{1o} - P_{1g} = P_{1cog}; P_{2o} - P_{2g} = P_{2cog};$$

$S_{1o} + S_{1g} = 1; S_{2o} + S_{2g} = 1.$ with the following initial and boundary conditions:

$$P_{1o}(x,0) = P^H_{1o}(x), P_{2o}(x,0) = P^H_{2o}(x); \quad (10)$$

$$P_{1g}(x,0) = P^H_{1g}(x), P_{2g}(x,0) = P^H_{2g}(x); \quad (11)$$

$$S_{1o}(x,0) = S^H_{1o}(x), S_{2o}(x,0) = S^H_{2o}(x); \quad (12)$$

$$S_{1g}(x,0) = S^H_{1g}(x), S_{2g}(x,0) = S^H_{2g}(x); \quad (13)$$

$$\begin{cases} -\frac{k_1}{\mu_o}\frac{\partial P_{1o}}{\partial x} = \alpha(P_A - P_{1o}); -\frac{k_1}{\mu_g}\frac{\partial P_{1g}}{\partial x} = \alpha(P_A - P_{1g}); x = 0; \\ -\frac{k_2}{\mu_o}\frac{\partial P_{2o}}{\partial x} = \alpha(P_A - P_{2o}); -\frac{k_2}{\mu_g}\frac{\partial P_{2g}}{\partial x} = \alpha(P_A - P_{2g}); x = 0; \end{cases} \quad (14)$$

$$\begin{cases} \frac{k_1}{\mu_o}\frac{\partial P_{1o}}{\partial x} = \alpha(P_B - P_{1o}); \frac{k_1}{\mu_g}\frac{\partial P_{1g}}{\partial x} = \alpha(P_B - P_{1g}); x = L; \\ \frac{k_2}{\mu_o}\frac{\partial P_{2o}}{\partial x} = \alpha(P_B - P_{2o}); \frac{k_2}{\mu_g}\frac{\partial P_{2g}}{\partial x} = \alpha(P_B - P_{2g}); x = L; \end{cases} \quad (15)$$

In the system of equations, the following functions need to be applied and utilized:

$$P_{1cog} = f_1(S_{1g}), P_{2cog} = f_2(S_{2g}); K_{1g} = f_3(S_{1g}); K_{2g} = f_4(S_{2g});$$

$$K_{1o} = f_5(S_{1g}); K_{2o} = f_6(S_{2g}); \rho_{1o} = const; \rho_{1g} = P_{1g}/RTZ;$$

$\rho_{2o} = const; \rho_{2g} = P_{2g}/RTZ$. Here:

R – universal gas constant;

T – temperature;

Z – gas compressibility factor.

In the equations and boundary conditions, the following notations are adopted:

The index *1* refers to variables related to the upper layer, and the index *2* refers to variables related to the lower layer.

l=o,g *l* – phase identification index: “o” – for the oil phase; “g” – for the gas phase.

q_o, q_g – well production rates for the lower layer;

δ – Dirac delta function. $\delta = \delta(x - x_i)$;

$P^H_{1o}, P^H_{2o}, P^H_{1g}, P^H_{2g}$ – initial pressure for oil and gas in the upper and lower layers, respectively;

P_A, P_B – pressure at the right and left boundaries;

$$\alpha = \begin{cases} 0, closed\ boundary\ condition, \\ 1, mass\ exchange\ cond. \end{cases}$$

P_{1cog}, P_{2cog} – capillary pressure in the oil-gas system.

$$P_{1o} - P_{1g} = P_{1cog}(S_{1o}, S_{1g}), P_{2o} - P_{2g} = P_{2cog}(S_{2o}, S_{2g});$$

$S^H_{1o}, S^H_{2o}, S^H_{1g}, S^H_{2g}$ – saturation, respectively for oil and gas in the upper and lower layers;

k_1, k_2 – absolute permeability coefficient**;**

K_l – relative phase permeability coefficients for phase *l*;

m – porosity of the formation;

μ_l – viscosity for phase *l*;

ρ_l – *l* – density of phase;

k_{Π} – ermeability coefficient of the low-permeability layer;

h_1, h_2, h_{Π} – thicknesses of the layers.

In the boundary value problem (9)–(15), we proceed to dimensionless variables using the following formulas. We consider the oil filtration process in a two-phase oil-water system within a heterogeneous double-layer porous medium that includes low-permeability intermediate layers. The focus is on the effect of their dynamic interaction. Due to the low permeability of the intermediate layers, fluid movement occurs only in the vertical direction (Fig. 2).

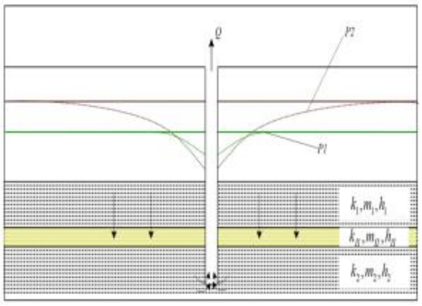

Fig. 2. The filtration process for a two-phase oil-water system in a double-layer porous medium with dynamic interaction

When designing and analyzing multiphase filtration processes in the oil-water system of multilayer oil fields, it is necessary to take into account the existence of hydrodynamic connectivity between the layers. If both layers possess the same reservoir properties, then the two-phase filtration problem in the oil-water system can be represented in one- or two-dimensional form.

In the study of two-phase filtration processes—such as in oil-water or oil-gas systems—key performance indicators of reservoir development include the reservoir pressure function and the saturation level within the formation. Furthermore, the degree to which the relative permeability coefficient has been accurately determined from experiments also plays a significant role. Since the mathematical model is nonlinear with respect to these indicators and the relative permeability coefficient, solving it numerically becomes significantly more complex.

Assume that the reservoir properties are the same in both layers. In that case, the two-phase filtration problem in the oil-water system can be described by a two-dimensional mathematical model in the form of a system of nonlinear parabolic-type differential equations. For simplicity, in the given boundary value problem, we consider the region as a square domain, i.e. $G = \{0 < x < L, 0 < y < L\}$. When a low-permeability intermediate layer exists between the two oil-bearing layers, the mathematical model of the problem in both oil layers is described by the following system of coupled nonlinear parabolic-type differential equations.

$$\begin{cases}\frac{\partial}{\partial x}\left[\lambda_{1o}\frac{\partial P_{1o}}{\partial x}\right]+\frac{\partial}{\partial y}\left[\lambda_{1o}\frac{\partial P_{1o}}{\partial y}\right]=m\rho_o\frac{\partial S_{1o}}{\partial t}-\frac{\rho_o k_{\Pi}}{h_1 h_{\Pi}\mu_o}\left(P_{2o}-P_{1o}\right),\\ \frac{\partial}{\partial x}\left[\lambda_{1w}\frac{\partial P_{1w}}{\partial x}\right]+\frac{\partial}{\partial y}\left[\lambda_{1w}\frac{\partial P_{1w}}{\partial y}\right]=m\rho_w\frac{\partial S_{1w}}{\partial t};\\ \frac{\partial}{\partial x}\left[\lambda_{2o}\frac{\partial P_{2o}}{\partial x}\right]+\frac{\partial}{\partial y}\left[\lambda_{2o}\frac{\partial P_{2o}}{\partial y}\right]=m\rho_o\frac{\partial S_{2o}}{\partial t}+\frac{\rho_o k_{\Pi}}{h_2 h_{\Pi}\mu_o}\left(P_{2o}-P_{1o}\right)+Q_{2o};(16)\\ \frac{\partial}{\partial x}\left[\lambda_{2w}\frac{\partial P_{2w}}{\partial x}\right]+\frac{\partial}{\partial y}\left[\lambda_{2w}\frac{\partial P_{2w}}{\partial y}\right]=m\rho_w\frac{\partial S_{2w}}{\partial t};\\ S_{1o}+S_{1w}=1;\qquad S_{2o}+S_{2w}=1;\\ P_{1o}-P_{1w}=P_{1cow};\quad P_{2o}-P_{2w}=P_{2cow};\quad 0<x<L;\ 0<y<L;\ t>0.\end{cases}$$

It is solved under the following initial and boundary conditions[16]:

$$P_{1o}(x,y,0)=P_{1o}^H(x,y), P_{2o}(x,y,0)=P_{2o}^H(x,y); \quad (17)$$

$$P_{1w}(x,y,0)=P_{1w}^H(x,y), P_{2w}(x,y,0)=P_{2w}^H(x,y); \quad (18)$$

$$S_{1o}(x,y,0)=S_{1o}^H(x,y), S_{2o}(x,y,0)=S_{2o}^H(x,y); \quad (19)$$

$$S_{1w}(x,y,0)=S_{1w}^H(x,y), S_{2w}(x,y,0)=S_{2w}^H(x,y); \quad (20)$$

$$\begin{cases}-\frac{k}{\mu_o}\frac{\partial P_{1o}}{\partial x}=\alpha(P_{Ao}-P_{1o}); x=0;\\ -\frac{k}{\mu_w}\frac{\partial P_{1o}}{\partial y}=\alpha(P_{Ao}-P_{1o}); y=0;\\ \frac{k}{\mu_o}\frac{\partial P_{1o}}{\partial x}=\alpha(P_{Ao}-P_{1o}); x=L;\\ \frac{k}{\mu_w}\frac{\partial P_{1o}}{\partial y}=\alpha(P_{Ao}-P_{1o}); y=L.\end{cases} \quad (21)$$

$$\begin{cases}-\frac{k}{\mu_w}\frac{\partial P_{1w}}{\partial x}=\alpha(P_{Aw}-P_{1w}); x=0;\\ -\frac{k}{\mu_w}\frac{\partial P_{1w}}{\partial y}=\alpha(P_{Aw}-P_{1w}); y=0;\\ \frac{k}{\mu_w}\frac{\partial P_{1w}}{\partial x}=\alpha(P_{Aw}-P_{1w}); x=L;\\ \frac{k}{\mu_w}\frac{\partial P_{1w}}{\partial y}=\alpha(P_{Aw}-P_{1w}); y=L;\end{cases} \quad (22)$$

$$\begin{cases}-\frac{k}{\mu_o}\frac{\partial P_{2o}}{\partial x}=\alpha(P_{Ao}-P_{2o}); x=0;\\ -\frac{k}{\mu_w}\frac{\partial P_{2o}}{\partial y}=\alpha(P_{Ao}-P_{2o}); y=0;\end{cases} \quad (23)$$

$$\begin{cases}\frac{k}{\mu_o}\frac{\partial P_{2o}}{\partial x}=\alpha(P_{Ao}-P_{2o}); x=L;\\ \frac{k}{\mu_w}\frac{\partial P_{2o}}{\partial y}=\alpha(P_{Ao}-P_{2o}); y=L.\end{cases}$$

$$\begin{cases}-\frac{k}{\mu_w}\frac{\partial P_{2w}}{\partial x}=\alpha(P_{Aw}-P_{1w}); x=0;\\ -\frac{k}{\mu_w}\frac{\partial P_{2w}}{\partial y}=\alpha(P_{Aw}-P_{2w}); y=0;\\ \frac{k}{\mu_w}\frac{\partial P_{2w}}{\partial x}=\alpha(P_{Aw}-P_{1w}); x=L;\\ \frac{k}{\mu_w}\frac{\partial P_{2w}}{\partial y}=\alpha(P_{Aw}-P_{2w}); y=L.\end{cases} \quad (24)$$

Here, the values h_x, k_x, P^H refer to the thicknesses of the layers, permeability coefficients, and characteristic values of pressure. Using these formulas, we formulate the following dimensionless boundary value problem for the oil-gas and oil-water systems.

$$\begin{cases}\frac{\partial}{\partial x}\left[K_{1o}k_1\left(\frac{\partial P_{1o}}{\partial x}\right)\right]=\frac{\partial}{\partial \tau}(1-S_{1g}),\\ \frac{\partial}{\partial x}\left[R_s K_{1o}k_1\left(\frac{\partial P_{1o}}{\partial x}\right)\right]+\frac{\mu_{1o}}{\rho_{1o}\mu_{1g}}\frac{\partial}{\partial x}\left[K_{1g}k_1\rho_{1g}\left(\frac{\partial P_{1g}}{\partial x}\right)\right]=\\ =\frac{\partial}{\partial \tau}\left[R_s(1-S_{1g})+\frac{\rho_{1g}}{\rho_{1o}}S_{1g}\right]-\frac{\rho_{1g}\mu_{1o}}{\rho_{1o}\mu_{1g}}\frac{k_{\Pi}L^2}{h_x h_1 h_x h_{\Pi}}\left(P_{2g}-P_{1g}\right);\\ \frac{\partial}{\partial x}\left[K_{2o}k_2\left(\frac{\partial P_{2o}}{\partial x}\right)\right]=\frac{\partial}{\partial \tau}(1-S_{2g})+\frac{\mu_{2o}L^2}{k_x\rho_{2o}P^H};\\ \frac{\partial}{\partial x}\left[R_s K_{2o}k_2\left(\frac{\partial P_{2o}}{\partial x}\right)\right]+\frac{\mu_{2o}}{\rho_{2o}\mu_{2g}}\frac{\partial}{\partial x}\left[K_{2g}k_2\rho_{2g}\left(\frac{\partial P_{2g}}{\partial x}\right)\right]=\\ =\frac{\partial}{\partial \tau}\left[R_s(1-S_{2g})+\frac{\rho_{2g}}{\rho_{2o}}S_{2g}\right]+\frac{\rho_{2g}\mu_{2o}}{\rho_{2o}\mu_{2g}}\frac{k_{\Pi}L^2}{h_x h_2 h_x h_{\Pi}}\left(P_{2g}-P_{1g}\right)+\frac{\mu_{2o}L^2}{k_x\rho_{2o}P^H}Q_2;\\ P_{1o}-P_{1g}=P_{1cog};\quad P_{2o}-P_{2g}=P_{2cog};\\ S_{1o}+S_{1g}=1;\qquad S_{2o}+S_{2g}=1.\end{cases} \quad (25)$$

we write the same equation in the oil-water formulation

$$\begin{cases}\frac{\partial}{\partial x}\left[K_o\left(cP_{1o}+(1-c)\right)\frac{\partial P_{1o}}{\partial x}\right]+\frac{\partial}{\partial x}\left[K_w\left(\frac{\partial P_{1o}}{\partial x}-\frac{\partial P_{1cow}}{\partial x}\right)\right]+\\ \frac{\partial}{\partial y}\left[K_o\left(cP_{1o}+(1-c)\right)\frac{\partial P_{1o}}{\partial y}\right]+\frac{\partial}{\partial y}\left[K_w\left(\frac{\partial P_{1o}}{\partial y}-\frac{\partial P_{1cow}}{\partial y}\right)\right]=\\ c\frac{\partial}{\partial t}(S_{1o}P_{1o})+(1-c)\frac{\partial S_{1o}}{\partial \tau}+\frac{\mu_w}{\mu_o}\frac{\partial(1-S_{1o})}{\partial \tau}-\frac{\rho_o k_{\Pi}L^2}{kh_1 h_{\Pi}}\left(P_{2o}-P_{1o}\right),\end{cases}$$

$$\frac{\partial}{\partial x}\left[K_o\left(cP_{2o}+(1-c)\right)\frac{\partial P_{2o}}{\partial x}\right]+\frac{\partial}{\partial x}\left[K_w\left(\frac{\partial P_{2o}}{\partial x}-\frac{\partial P_{2cow}}{\partial x}\right)\right]+$$

$$\frac{\partial}{\partial y}\left[K_o\left(cP_{2o}+(1-c)\right)\frac{\partial P_{2o}}{\partial y}\right]+\frac{\partial}{\partial y}\left[K_w\left(\frac{\partial P_{2o}}{\partial y}-\frac{\partial P_{2cow}}{\partial y}\right)\right]= \quad (26)$$

$$c\frac{\partial}{\partial t}(S_{2o}P_{2o})+(1-c)\frac{\partial S_{2o}}{\partial \tau}+\frac{\mu_w}{\mu_o}\frac{\partial(1-S_{2o})}{\partial \tau}+\frac{\rho_o k_{\Pi}L^2}{kh_2 h_{\Pi}}\left(P_{2o}-P_{1o}\right)+Q_{2o}$$

From this system of equations, it is evident that the equations are only in relation to the oil pressure function. Therefore, they must be solved with respect to the pressure functions in a coupled manner at each time interval. In this case, the initial and boundary conditions mentioned above are applied only to oil, i.e.:

The solution of this boundary value problem is carried out by applying the Thomas algorithm (progonka method) developed for the finite difference system, and by using the quasi-linear method for the nonlinear terms in the system of equations. At each time step, the solution is obtained through iteration with respect to the pressure function and the viscosity coefficients.

Here, A_i, B_i, C_i and A_i', B_i', C_i' are the Thomas algorithm (progonka) coefficients, which are determined using the following formulas:

$$A_i=\frac{c_i\left(b_i'-a_i'A_{i-1}'\right)}{R_i};\quad B_i=\frac{c_i'\left(a_iB_{i-1}+d_i\right)}{R_i};$$

$$A_i'=\frac{\left(b_i-a_iA_{i-1}\right)c_i'}{R_i};\quad B_i'=\frac{c_i\left(a_i'B_{i-1}'+d_i'\right)}{R_i};$$

$$C_i=\frac{\left(a_iB_{i-1}+d_i\right)\left(a_i'C_{i-1}'+f_i'\right)+\left(a_iC_{i-1}+f_i\right)\left(b_i'-a_i'A_{i-1}'\right)}{R_i};$$

$$C_i'=\frac{\left(a_i'B_{i-1}'+d_i'\right)\left(a_iC_{i-1}+f_i\right)+\left(a_i'C_{i-1}'+f_i'\right)\left(b_i-a_iA_{i-1}\right)}{R_i};$$

$$R_i=\left(b_i-a_iA_{i-1}\right)\left(b_i'-a_i'A_{i-1}'\right)-\left(a_iB_{i-1}+d_i\right)\left(a_i'B_{i-1}'+d_i'\right);$$

$i=1,2,\ldots,N-1.$

The mathematical model of the processes occurring in oil-bearing layers connected through a dynamically interacting low-permeability interlayer, as described above,

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is highly complex. Solving it is only feasible using numerical methods.

The corresponding computational scheme is also intricate and consists of two main stages, primarily based on finite difference and iterative methods. It is implemented in the following sequence:

1. Assigning initial data values:
Number of time iterations – *nt*;
Permeability coefficient value – *k*;
Porosity coefficient value – *m*;
Reservoir length – *L*;
Oil viscosity coefficient – μ;
Pressure of oil in the reservoir – *P*;
Saturation coefficients of oil and water – S_o, S_w;

2. Time iteration cycle: *k=1…nt;*

3. **First Stage.** This stage involves performing calculations at the *k + 0.5* time layer. For each value of *j = 1…n−1*, the following steps are executed:

3.1. The coefficients of the finite difference equations a_i, b_i, c_i, d_i, f_i and $a'_i, b'_i, c'_i, d'_i, f'_i$ are calculated for *i = 1…n−1*;

3.2. The progonka (sweep) coefficients A_0; B_0; C_0; A'_0; B'_0; C'_0 are determined from the left boundary conditions.

3.3. The progonka (sweep) coefficients A_i; B_i; C_i; A'_i; B'_i; C'_i *(i=1,n-1)* are computed from left to right.

3.4. $P_{1i,j}$ and $P_{2i,j}$ are determined from the right boundary condition;

3.5. $P_{1i,j}$ and $P_{2i,j}$ *k+0.5* are computed at the *k + 0.5* time layer;

3.6. The iterative process is checked at this time layer.

$$\left|P_{1oi,j}^{(r)} - P_{1oi,j}^{(r-1)}\right| \le \varepsilon_p \text{ and } \left|P_{2oi,j}^{(r)} - P_{2oi,j}^{(r-1)}\right| \le \varepsilon_p.$$

If this iteration condition is satisfied, that is, the following conditions hold, proceed to step 3.7; otherwise, return to step 3.1.

Here,

$P_{1o}^{(r)}, P_{1o}^{(r-1)}, P_{2o}^{(r)}, P_{2o}^{(r-1)}$ – pressure functions with two closely related values (*r* – the currently calculated value, *r–1* – the previously calculated value; the initial value is taken from the starting condition, and the reference value is taken from the previous iteration);

P_{1o}, P_{2o} – pressure functions;

ε_p – iteration accuracy.

3.7.P_{1o}, P_{2o} the oil pressure in the first and second layers is calculated with sufficient accuracy.

3.8. P_{1w}, P_{2w} the water pressure in the first and second layers is determined using the following formulas:

$$P_{1w} = P_{1o} - P_{cow}; P_{2w} = P_{2o} - P_{cow}$$

3.9. The first and fourth equations of the system are approximated to calculate the water saturation coefficients S_{1w} and S_{2w}.

3.10. Since the system of equations is nonlinear with respect to the saturation functions, an iterative process is applied. This iterative process continues until the following conditions are satisfied.

$$\left|S_{1wi,j}^{(r)} - S_{1wi,j}^{(r-1)}\right| \le \varepsilon_s \text{ and } \left|S_{2wi,j}^{(r)} - S_{2wi,j}^{(r-1)}\right| \le \varepsilon_s.$$

Here

$S_{1w}^{(r)}, S_{1w}^{(r-1)}, S_{2w}^{(r)}, S_{2w}^{(r-1)}$ – pressure functions S_{1w}, S_{2w}, which have two close values (where *r* is the currently calculated value, *r-1* is the previously calculated value, with the zero-th value taken from the initial condition, and the subsequent values are taken from the previous iteration).

ε_s- iteration accuracy.

3.11.S_{1w}, S_{2w} In the first and second layers, the water saturation is calculated with sufficient accuracy using the following formula:

$$S_{1o} = 1 - S_{1w}; S_{2o} = 1 - S_{2w}$$

3.12. If the iterative process is satisfied, proceed to step 3.13**;** otherwise, return to step 3.1**.**

4. **Second stage.** In the second stage, the above calculations are repeated for the *k+1* time layer in the same way, i.e., from step 3.1 to 3.12.

5. The solutions obtained at time layer *k+1* serve as the initial values for the next step *k+2*.

6. The numerical results are displayed on the screen in the form of tables and 3D graphs.

7. End of the time iteration loop. If the specified number of iterations matches the time loop cycle, the program terminates; otherwise, it returns to step 2.

The developed algorithm can be easily adapted for two- and three-point finite difference equations and, in addition, can also be applied to other multiphase filtration problems, such as the "oil-gas" and "oil-gas-water" systems.

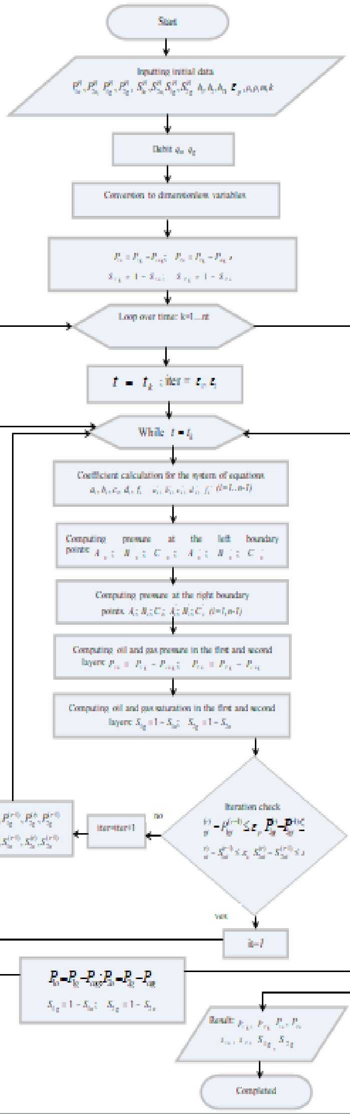

Fig. 3. Block diagram of the algorithm for solving the boundary value problem of the filtration process for a two-phase oil-gas system with mutual dynamic interaction in a two-layer medium

Computational experiments

Based on the numerical model, software has been developed to calculate the main indicators of oil field development in a dynamically connected two-layer system. The software consists of modules for inputting initial data, calculating indicators, and displaying the computation results. The calculation results of the indicators are presented in graphical form.

From the beginning of oil field development, numerical simulation experiments were conducted to analyze the distribution of oil pressure in the reservoir over a period of 720 days. In these figures, the first two graphics show the 3D visualization of oil pressure distribution in the upper and lower layers of the reservoir.

It is clearly observed from the simulation results that the pressure in the upper layer decreases very slowly. This indicates that the flow of oil from the upper to the lower layer is minimal. This limited transfer is due to the very low permeability coefficient of the intermediate layer separating the two reservoir zones.

The contour plot in the second row of the figures illustrates the distribution of oil pressure in the lower layer. The second graph in the row shows the variation of oil pressure along a section at different permeability coefficient values in the wells located in the lower layer.

The material balance equation was used to evaluate the numerical results obtained during the computational experiments. Table 1 presents the variation of the average reservoir pressure over time, where the average values calculated using the finite difference equation and the material balance solutions are provided. The numerical solution obtained by computer was compared with the value calculated using the material balance method, and relative error values were also given to assess accuracy[18].

Table 1

Comparison of the average reservoir pressure using two methods

Number of days	Oil pressure in the well, atm	Average numerical solution obtained by computer	Average pressure calculated using material balance	Relative error (%)
40	284,82	299,04	299,76	0,0380
120	281,79	298,92	299,28	0,1230
240	279,66	298,75	298,56	0,2776
480	275,33	295,32	297,12	0,6060
720	273,03	292,92	295,68	0,9387

One of the methods for analyzing the practical convergence of the finite difference solution to the differential problem is to refine the time grid step. If successive reductions in the time step do not lead to significant changes in the results at the same spatial grid points, it can be concluded that convergence is present. For this purpose, calculations were carried out at different values of the time grid step over $\Delta\tau$ stages, and the results are presented in Table 2, confirming convergence with respect to time.

Table 2

Variation of the average reservoir pressure and the well pressure over time at different time steps

Days		80	160	240	320	400
$\tau = 2$	$P_{Initial}$	0,9428	0,9365	0,9322	0,9284	0,9247
	P_{Final}	0,9976	0,9951	0,9925	0,9899	0,9871

Days		480	560	640	720
$\tau = 2$	$P_{Initial}$	0,9211	0,9174	0,9138	0,9101
	P_{Final}	0,9844	0,9818	0,9791	0,9764

In Table 2, the results of the calculations based on formulas (6) and (7) are presented, demonstrating that the calculated pressure values are in close agreement.

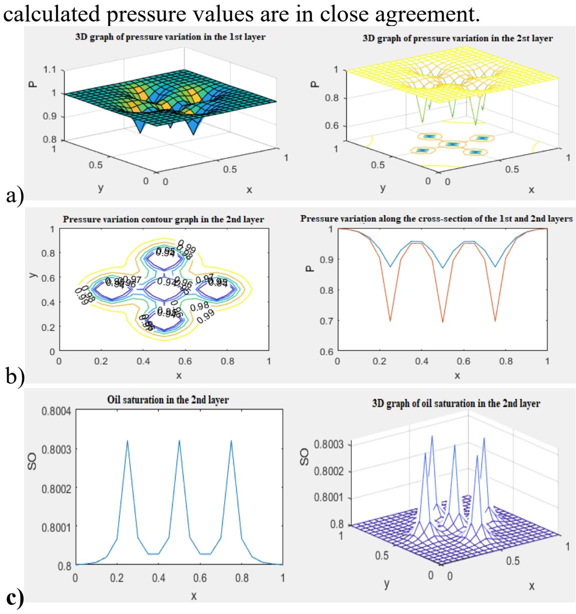

Fig. 4. Pressure distribution graphs in the upper and lower oil layers
(k_1=0.1 d.; k_2=0.1 d.; k_p=0.002 d.; μ=4 sP; m=0.1)

In the a) graph, a 3D plot of pressure distribution in the reservoir is observed when five wells are symmetrically located. In the b) graph, a 3D plot of pressure distribution in the second layer is shown. The c) graph presents the contour plot of pressure distribution, while the fourth graph illustrates pressure variation along a cross-section. The fifth and sixth graphs display oil saturation in the second layer — in cross-section and 3D views, respectively. In this case, the following parameter values were used: k_1=0.1 d.; k_2=0.1 d.; k_p=0.002 d.; μ=4 sP; m=0.1.

3. Research results

The main features of the software complex for analyzing and forecasting the filtration process in oil fields include solving a boundary value problem based on a mathematical model to determine the key indicators of oil filtration in reservoir layers and conducting computational experiments with visualization. The developed software complex titled "*Modeling and Visualization of Computational Experiments* *for Determining Key Performance Indicators of Oil Field Development*" is created based on the above-mentioned mathematical model, numerical methods, and solution algorithms.

In the process of oil and gas extraction, automating the solution of fluid and gas filtration boundary value problems in porous media allows for the determination of hydrodynamic parameters of reservoirs, speeds up the design of oil and gas fields, and enhances the analysis and forecasting of the filtration process. Therefore, developing and effectively using automated systems for studying the oil filtration process in porous media is essential for solving various problems specific to the field.

The development of a software complex that solves the boundary value problem of oil filtration in porous media and ensures the accuracy of computational results provides significant convenience for users in determining key indicators of the filtration process in oil reservoir layers. This software complex enables rapid determination of hydrodynamic parameters during oil or gas extraction in reservoir systems, playing an important role especially in the design, analysis, and forecasting of oil and gas fields.

Automating the solution of boundary value problems based on a mathematical model to identify key indicators in the filtration process of fluids and gases in fractured, heterogeneous underground porous layers contributes to the creation of specialized software for solving various related problems. To determine the main indicators of the oil or gas filtration process in oil and gas reservoir layers, the solution of a boundary value problem based on a mathematical model is carried out, along with computational experiments visualized on a computer[17].

The software complex makes it possible to conduct computational experiments on key indicators for a two-dimensional, three-layer oil system characterized by weak dynamic interaction through a semi-permeable layer in a fractured, heterogeneous filtration domain of both simple and complex structure. The developed software not only serves as an interactive tool for analyzing data on a computer but also provides the ability to conduct simulations, analyze processes, and make necessary decisions — whether brief or comprehensive — during the modeling and forecasting stages.

Computational experiments based on the boundary value problem of filtration for a two-phase oil-water system in a two-layer porous medium with mutual dynamic interaction show that the smaller the value of the permeability parameter of the weakly permeable layer, the less pressure drop occurs in the upper and lower layers. This pressure drop has a greater impact on the areas near the well in the second layer. The developed model, algorithm, and software can be used to analyze and forecast the development of multilayer oil and gas fields with mutual dynamic interaction.

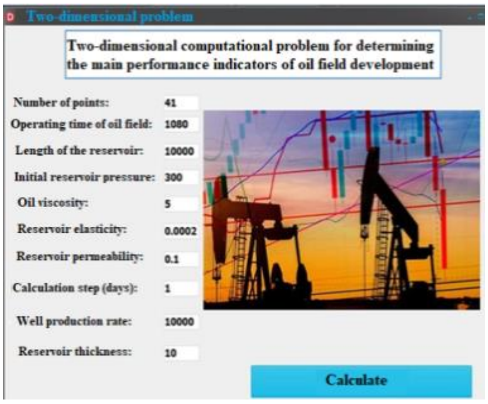

Fig. 5. User interface of the program for solving the two-dimensional filtration problem for a two-phase oil-water system

Implemented work: The developed numerical models and software tools for the filtration process of two-phase oil-water and oil-gas systems in a porous medium with mutual dynamic interaction enable the study and development of oil fields.

Scientific and technical significance: The developed numerical model and algorithm, along with the software complex for calculating the key performance indicators of oil field development, can be used for analysis, design, and the development of oil fields with mutual dynamic interaction.

4. Conclusion

The developed mathematical models and software tools provide a comprehensive framework for analyzing unsteady two-phase filtration processes — oil-water and oil-gas systems — in porous media. Based on the fundamental equations and principles of unsteady filtration theory, these models have been applied to both single- and two-layer structures to assess the hydrodynamic behavior of oil reservoirs. For systems with mutually dynamic interacting layers, a two-dimensional mathematical model and corresponding boundary conditions were formulated, leading to the development of efficient computational algorithms. These algorithms significantly improve the accuracy and speed of calculating reservoir pressure and saturation levels[18].

Moreover, a finite difference solution algorithm based on the method of directional splitting was implemented, and specialized software was developed accordingly. This software enables real-time monitoring of the filtration process, visualization of numerical results, and execution of interactive computational experiments. Overall, the proposed model, algorithm, and software package serve as an effective tool for the design, analysis, and forecasting of oil field development.

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This research work provided an opportunity to explore two-phase filtration processes through a combination of theoretical and practical approaches, enabling the development of mathematical models and computational experiments. By utilizing advanced methods and technologies, efficient and accurate results were achieved in the design of numerical algorithms and software tools. In

particular, the solutions developed for modeling hydrodynamic processes in multilayer systems with mutual dynamic interaction have practical significance for analyzing and forecasting oil field development. I extend my sincere appreciation to all specialists who contributed valuable scientific guidance, technical assistance, and essential data throughout the course of this study.

Their support played an important role in successfully completing this research and achieving its intended goals.

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