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Theoretical approaches to cutting force determination: a review

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Abstract: This article provides a comprehensive review of the main theoretical approaches to determining cutting forces that have been proposed in various scientific studies. Each method is analyzed in terms of its fundamental principles, advantages, limitations, and suitability for application in industrial machining processes. Particular attention is given to the comparison between theoretical calculations and experimental results, which allows for an objective evaluation of the accuracy and reliability of the reviewed models. The paper discusses both classical theories, such as those developed by Merchant and Zorev, and more advanced concepts proposed by researchers like Klocke, Davim, and Astakhov, including energy-based and wave-dependent models. By exploring a wide range of equations and analytical methods, the article highlights how cutting force is influenced by factors such as material properties, cutting speed, depth of cut, and tool geometry. The findings of this review contribute to a better understanding of cutting mechanisms and can be effectively used to improve the prediction of cutting forces, optimize machining parameters, reduce tool wear, and enhance the quality of machined components.

Keywords: machining, cutting force, theoretical methods, cutting force calculation, experimental comparison

1. Introduction

During the cutting process, resistance forces act on the cutting edge of the tool, opposing its motion along the trajectory of the relative working movement. The resultant of these forces is referred to as the cutting force. The study of cutting force is of significant interest, as its values are essential for calculating the mechanisms of feed and primary motion in machine tools. Knowledge of the cutting force enables regulation of the cutting process, including control of machine power consumption, assessment and adjustment of the temperature in the cutting zone, prevention of vibrations, and other related aspects.

Therefore, the considerable interest of researchers in the prediction and analysis of cutting forces is explained by the high relevance of this issue for improving the efficiency of machining processes, optimizing technological parameters, and reducing tool wear [1, 2].

In numerous studies dedicated to cutting force investigation, two main methods for determining its values are commonly used: theoretical and experimental. Both approaches have their own advantages and limitations. Researchers do not rely on a single method but continue to explore new ways of measuring cutting forces. This article focuses primarily on theoretical (analytical) methods for determining cutting force. A wide range of theoretical approaches has been developed for evaluating cutting forces. The present work emphasizes the analysis of existing theoretical methods, their development and practical application in industrial settings, as well as their comparison with experimental results.

2. Research methodology

2.1. Merchant's Circle Diagram and Zorev's Theory

The first theoretical studies on the determination of cutting forces are described in the works of Merchant [3, 4]. In his research, Merchant decomposed the resultant cutting force into several vector components, whose directions are inscribed within a circle. This graphical representation became known as the "Merchant's Circle Diagram" (Fig. 1).

As can be seen, the resultant force R is resolved into the friction force F between the tool and the chip, and the normal force N. The angle μ between the vectors N and R is called the friction angle. On the shear plane, the force R is further decomposed into the shear force F_S, which, according to Merchant, is spent on shearing the metal, and the normal force F_N, which exerts a compressive load on the shear plane.

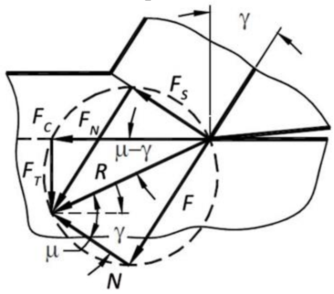

Fig. 1. Merchant's Circle Diagram [3]

Additionally, the force R is resolved along the direction of tool movement into the cutting force F_C and the thrust force F_T acting on the tool. The determination of the cutting force is based on calculating the shear force F_S:

$$F_s = \frac{\tau_y A_c}{\sin\varphi} \quad (1)$$

where τ_y is the shear stress of the workpiece material; φ is the shear angle; and A_c is the cross-sectional area of the undeformed chip.

According to Fig. 1, the cutting force is determined by the following formula:

$$F_c = \frac{F_s \cos(\mu-\gamma)}{\cos(\varphi+\mu-\gamma)} \quad (2)$$

Taking into account equation (1), the final expression for the cutting force can be written as:

$$F_c = \frac{\tau_y A_c \cos(\mu-\gamma)}{\sin\varphi\cos(\varphi+\mu-\gamma)} \quad (3)$$

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A similar calculation scheme for determining the cutting force is presented by N.N. Zorev [5–6]:

$$R_z = \tau_y \frac{ab}{\sin\varphi}\frac{\cos\omega}{\cos(\varphi+\omega)} \quad (4)$$

where R is the chip formation force; τ_y is the shear stress of the workpiece material; *a* is the shear thickness; *b* is the shear width; and ω is the angle of action, i.e., the angle between the force vector R and the cutting velocity vector *v* (Fig. 2).

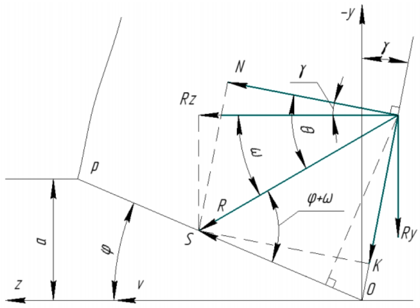

Fig. 2. Zorev's scheme: R – chip formation force; R_z – main (tangential) component of the chip formation force; R_y – radial component of the chip formation force; *a* – shear thickness; ω – angle of action; φ– shear angle; γ – rake angle [5]

It should be noted that neither Merchant's method nor Zorev's method accounts for tool wear and friction on the tool's rake face during the cutting process. When applying these methods in industrial settings, difficulties arise in determining the shear stress τ_y of the workpiece material, the friction angle μ, and the shear angle φ. In his works, Merchant provides an equation for determining the shear angle, which is expressed as follows:

$$\varphi = \arctan\left(\frac{r\cos\gamma}{1-r\sin\gamma}\right) \quad (5)$$

where γ is rake angle; r - chip thickness ratio:

$$\mathrm{r} = \frac{t_o}{t_c} \quad (6)$$

where t_o is the chip thickness before deformation, and t_c is the chip thickness after deformation.

Although the equation defining the shear angle appears to provide a complete description of the cutting process, it does not take into account the physical and mechanical properties of the machined material. The shear angle is determined solely based on changes in the geometric parameters of the chip (chip thickness) during cutting and the rake angle of the tool edge.

2.2. Development of Merchant's Theory

Merchant's method was further developed in the works of Kloke [7–9]. In his research, the author focuses on the theory of chip formation and investigates ideal cutting conditions, i.e., assumes an ideally sharp cutting edge. Kloke's circle diagram scheme is shown in Fig. 3.

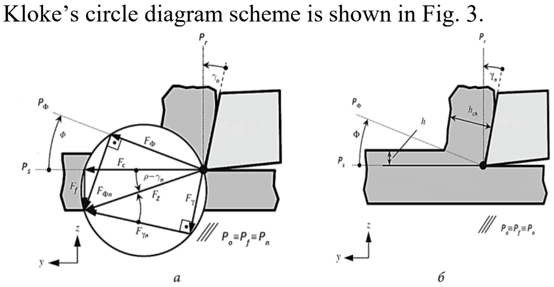

Fig. 3. Kloke's scheme: P_ϕ – shear plane trace; P_r – main plane trace; F_ϕ – shear force; F_c – cutting force; F_f – feed force; F_z – resultant force; Φ – shear angle; ρ – friction angle; h – cutting depth [7]

It is assumed that the shear angle is a function of the shear stress τ_ϕ in the shear plane:

$$\tau_\phi = \frac{|F_\phi|}{A_\phi}; \; A_\phi = b\frac{h}{\sin\Phi} \quad (7)$$

where A_ϕ is the cross-sectional area of the undeformed chip; h is the cutting depth (Fig. 3b); and b is the width of the tool.

The shear force F_ϕ can be expressed through the resultant force F_z as follows:

$$|F_\phi| = |F_z|\cos(\Phi+\rho-\gamma_n) \quad (8)$$

The resultant force F_z can be expressed using all of the above as follows:

$$|F_z| = \frac{\tau_\phi}{\sin\Phi\cos(\Phi+\rho-\gamma_n)}bh \quad (9)$$

The relationship between the cutting force F_c and the resultant force F_z:

$$|F_c| = |F_z|\cos(\rho-\gamma_n) \quad (10)$$

Finally, the equation for determining the cutting force proposed by Kloke takes the following form:

$$|F_c| = \frac{\cos(\rho-\gamma_n)\cdot\tau_\phi}{\sin\Phi\cos(\Phi+\rho-\gamma_n)}bh \quad (11)$$

where h is the cutting depth; b is the tool width; Φ is the shear angle; ρ is the friction angle; γ is the rake angle; and τ_ϕ is the shear stress.

In Kloke's equations, a calculation scheme similar to those of Merchant and Zorev is observed, with the main difference being the notation of the parameters used in the equations.

2.3. Energy-based method for determining cutting force

The energy-based approach to determining the cutting force was proposed by Davim [10–11], and his calculations take tool wear into account, which brings the computations closer to the real machining process. The energy balance within the system is expressed as follows:

$$P_c = F_c v = P_{pd} + P_{fR} + P_{fF} + P_{ch} \quad (12)$$

from which the cutting force is calculated as:

$$F_c = \frac{P_{pd}+P_{fR}+P_{fF}+P_{ch}}{v} \quad (13)$$

where P_{pd} is the energy consumed for the plastic deformation of the removed layer; P_{fR} is the energy consumed due to the interaction between the tool and the chip; P_{fF} is the energy consumed due to the interaction between the tool and the workpiece; and P_{ch} is the energy consumed for the formation of new surfaces.

As follows from the calculations, Davim's method involves summing all the energy components when determining the cutting force. Although the calculations provide a detailed description of the cutting process, this method cannot be practically applied in real conditions due to the labor-intensive nature of computing all energy components.

In his works [10–11], the author also compares experimental and theoretical values of cutting force determination during turning of AISI E52100 steel at a cutting speed v=90 m/min, feed S=0.2 mm/rev, and cutting depth of 2 mm. The discrepancy between calculated and experimental values is approximately 6% (998 N and 940 N, respectively).

2.4. Astakhov's Theory

A fundamentally different approach to determining the cutting force was proposed and experimentally confirmed by V. Astakhov [12–13]. He states that the cutting force exhibits a wave-like dependence on thermal energy and deformation energy. The underlying equation is given as:

$$dW_{in} = dA + dQ \tag{14}$$

where dW_{in} is the internal energy; dA is the mechanical energy supplied from outside; and dQ is the thermal energy dissipated in the system.

A micro-volume of the machined material located at point 2 along the tool path (Fig. 4) is considered at the moment when the tool passes this point. According to the above equation, the change in internal energy dW_{in} within the micro-volume is equal to the sum of the mechanical work of the external forces dA, applied by the tool, and the residual thermal energy dQ, which penetrates into the current position from an equivalent micro-volume located at a neighboring position along the tool path (from point 1, see Fig. 4).

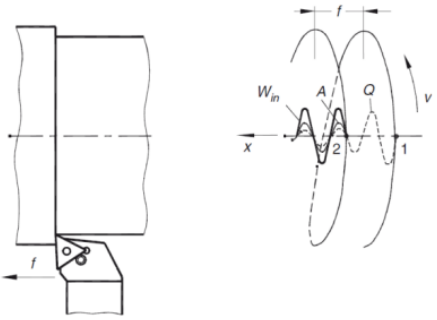

Fig. 4. External turning and tool path trajectory [11]

The residual heat dQ is positive because this heat flows into the micro-volume at point 2 (when the cutting tool reaches it) from a decaying heat source at point 1 (the previous tool position) if and only if the thermal conductivity velocity in the workpiece is equal to or greater than the velocity of the cutting tool movement in the feed direction. Thus, there is no contradiction with the laws of thermodynamics in the energy equation — heat first enters the less heated zone (point 2, see Fig. 4), and then the tool (as a heat source) moves to that zone, thereby causing heat release.

Astakhov provides experimental data. During turning of AISI 4140 steel at speeds ranging from 4 to 345 m/min, the range of cutting force was:

At a cutting depth t= 0.1 mm – 38 to 84 N (approximately 3.8 to 8.5 kgf);

At a cutting depth t=0.5 mm – 169 to 328 N (approximately 17 to 33 kgf);

At a cutting depth t=1.0 mm – 366 to 506 N (approximately 37 to 52 kgf).

Based on these data, a simple dependency is presented:

$$F_z = Ct^x S^y v^n \tag{15}$$

Astakhov then calculates the value of C using the experimental data, while the value of the exponent x is selected from a reference handbook. After substituting the values, the following result was obtained:

$$F_z = 53{,}94v^{-0{,}1} \text{ at } t = 0{,}1 \text{ mm} \tag{16}$$

$$F_z = 193{,}13v^{-0{,}1} \text{ at } t = 0{,}5 \text{ mm} \tag{17}$$

$$F_z = 427{,}79v^{-0{,}1} \text{ at } t = 1{,}0 \text{ mm} \tag{18}$$

The experimental values in the "v – Pz" coordinate system are shown in Fig. 5. Taking into account the wave-like nature of deformations, the author considers these points as sinusoidally periodic data, which can be mathematically represented as a wave-dependent function:

$$F_z = F_{z0} + F_{za} \sin\left[\frac{2\pi}{l_v}(v + v_{ph})\right] \tag{19}$$

where F_{z0} is the sinusoidal function; F_{za}, l_v, and v_{ph} are the amplitude, wavelength, and initial phase of the sinusoid, respectively.

As a result, taking Fig. 5 into account, the following expressions can be written:

$$F_z = 47 + 10\left[\frac{2\pi}{0{,}56}(0{,}050 + v)\right] \text{ at } t = 0{,}1 \text{ mm}; \tag{20}$$

$$F_z = 178 + 11\left[\frac{2\pi}{0{,}50}(0{,}049 + v)\right] \text{ at } t = 0{,}5 \text{ mm}; \tag{21}$$

$$F_z = 422 + 50\left[\frac{2\pi}{0{,}46}(0{,}045 + v)\right] \text{ at } t = 1{,}0 \text{ mm}; \tag{22}$$

It should be noted that at different cutting depths, the wave equations differ, and accordingly, the values of amplitudes and wavelengths also vary.

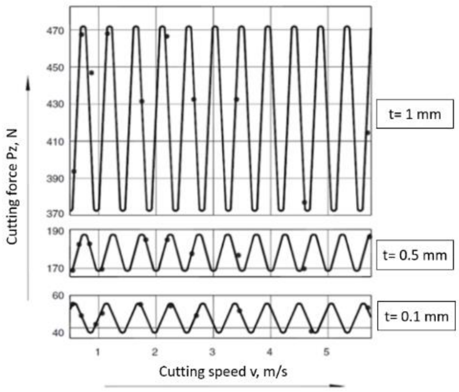

Fig. 5. Dependence of cutting force on cutting speed [11]

A similar wave-like dependence of the cutting force on cutting speed was observed in the studies by Krivoukhov [14] during the machining of titanium alloy. Experimental data were obtained using a VP-4-EI type electro-inductive three-component dynamometer. The results concerning the relationship between the components of the cutting force and the machining parameters during the turning of VT3 titanium alloy are presented in Table 1. Data modeling in the Matlab environment enables graphical representation of these dependencies (Fig.6).

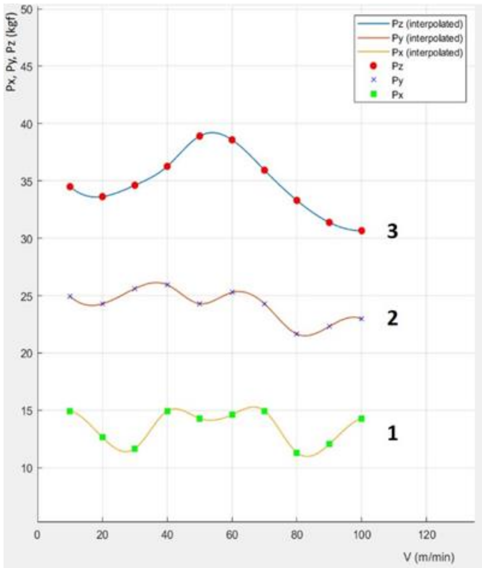

Fig. 6 Dependence of cutting force on cutting speed during turning of VT3 titanium alloy

Table 1

Experimental cutting force data obtained by Krivoukhov [14] during the turning of a titanium alloy

Cutting speed, m/min	Feed rate mm/rev	Cutting depth, mm	Components of the cutting force		
			P_z, kgf	P_x, kgf	P_y, kgf
10	0,11	1,0	34,50	14,95	24,95
20	0,11	1,0	33,63	12,64	24,29
30	0,11	1,0	34,62	11,65	25,61
40	0,11	1,0	36,27	14,95	25,94
50	0,11	1,0	38,91	14,29	24,29
60	0,11	1,0	38,58	14,62	25,28
70	0,11	1,0	35,94	14,95	24,29
80	0,11	1,0	33,30	11,32	21,65
90	0,11	1,0	31,98	12,04	22,31
100	0,11	1,0	30,66	14,29	22,97

As can be seen from Fig. 6, Astakhov's theory, which states that the cutting force has a wave-like dependence on thermal energy and deformation energy, is supported by the dependency graphs. However, it should be noted that the equations proposed by Astakhov were developed based on the machining of AISI 4140 steel, which limits their applicability to other materials. Additionally, the coefficients C, x, y, and n are not detailed, further restricting the practical use of these equations. Nevertheless, a thorough study of Astakhov's theory, in comparison with other research and supported by experimental data, can provide a comprehensive understanding of cutting forces.

3. Conclusion

All the aforementioned methods for determining cutting force are of significant interest not only for further research and practical implementation but also for expanding knowledge in the field of cutting force calculation. Continued study of this topic will not only improve the accuracy of cutting force prediction but also optimize machining processes, reduce tool wear, and enhance the quality of manufactured parts. Moreover, the application of modern calculation methods, including numerical modeling and experimental investigations, promotes the development of new approaches to process control and the selection of optimal machining parameters.

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