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Energy equations of a channel suspension-carrying flow with full account for the work of interphase interaction forces

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Abstract:

Channelized flows carrying suspended solids are a critical component of river and industrial fluid transport systems, where the behavior of suspended particles significantly influences flow dynamics and efficiency. In such systems, interactions between the fluid and sediment particles, influenced by forces such as drag and upwelling, play an important role in determining sediment suspension, entrainment, and transport. Accurate modeling of these interactions is necessary to predict flow behavior under a variety of conditions, but previous studies have often simplified these interactions by focusing on idealized cases of steady or uniform flow.

The paper presents a study of the movement of a channel suspension-carrying flow, with a comprehensive consideration of the work of interphase interaction forces. Understanding how suspended sediment particles are distributed across the flow depth is important for both theory and practice, especially for studying how river sediments are suspended, moved, and carried. However, many studies on unsteady and uniform movement don't fully match the real behavior of sediment-carrying flows in channels.

Keywords:

interphase interactions, suspension-carrying flow, unsteady motion, sediment transport, suspended particles, hydrodynamics

1. Introduction

Particles suspended in fluids usually move in channels with flows of carrier suspensions. These flows are considered essential for both industrial and natural processes. Understanding how these flows function is important. It affects the prediction of river sediment movement and erosion. In addition, knowledge of these flows is required to design slurry transport systems for mining and wastewater treatment. These flows are characterized by complex interactions between suspended particles and the surrounding fluid. These particles are distributed under the influence of forces such as drag, uplift, and gravity. These forces also play an important role in the movement of particles.

Turbulence is a key factor affecting sediment dynamics [1]. In their studies, [2] and [3] demonstrated that produced and dissipated in the flow the kinetic energy of turbulence directly affects sediment suspension. Other researchers [4] used a two-fluid model to analyze the mass and momentum exchange between the fluid and sediment phases. This highlighted the role of drag and the influence of particle inertia. Similarly, the vertical flow and sedimentation dynamics in open channel and boundary layer flows investigated by (Toorman, 2008.) and (Necker et al., 2001). They highlighted how turbulence and sediment concentration affect sediment suspension and resuspension.

Understanding particle interactions in turbulence is equally important. Studies by [7] in 2013 and more recently [8] examined the behaviour of particles in fibre suspensions, showing how turbulence-induced collisions and aggregation occur. Extended studies by [9] [10] highlighted how turbulence structures such as deformation zones influence collision frequencies and particle distributions. [11]

examined orientation distributions for non-spherical particles, demonstrating how these particles align differently in laminar and turbulent regimes, while [12] reinforced the importance of particle drag, especially in systems with different particle sizes.

Given these interactions considered above researches, to simulate particle-laden flows have been developed various numerical methods. Conserving volume by tracking particle fluxes through grid cells [13] presented an Eulerian approach, while [14] extended this to an Eulerian-Lagrangian framework. This capable of capturing near-wall interactions. [15] and [16] highlighted the need for appropriate turbulence closure models, especially in complex geometries. Elghobashi emphasized that the Lagrangian approach is best for one-way coupling, whereas the two-fluid model is suitable for two-way and four-way coupling scenarios.

The importance of specific interfacial forces varies depending on flow conditions. Studies in bubble-filled flows by [17] quantified drag, buoyancy, lift, and virtual mass. Chen found that drag and buoyancy were dominant, a finding also confirmed by [18] for open-channel flows. [19] examined flows with diverse solid concentrations, using a two-phase model with rheological properties and interphase slip effects, which is essential for accurately capturing momentum exchange in dense sediment flows. The combined effects of interphase forces on particle distribution were also investigated by [20], who applied particle-resolved simulations to study stress distributions on particle surfaces, an important factor for sediment suspension.

Additionally, sediment-laden flows are influenced by gravity and buoyancy forces. These forces play roles in particle sedimentation and resuspension. (Necker et al., 2001) conducted computational studies on gravity currents,

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examining particle settling and the potential for resuspension under changing flow conditions. [21] studied the motion of dust particles in protoplanetary disks, offering insights into particle-fluid friction and how it alters the flow. These concepts can be compared to sediment flows in river environments. [22] tested various solid-fluid interaction models for settling particles, highlighting the limitations of Lattice-Boltzmann approaches in capturing accurate force relationships, particularly when Archimedean principles are generalized for settling.

Accurate modeling of these flows requires considering the interfacial forces that control particle suspension, entrainment, and transport. However, previous studies on suspension flow in channels often focus on simplified cases, such as steady or uniform flows. They do not fully include the dynamic effects of interfacial forces in a complete physical model. This simplification makes it harder to predict particle behavior in real-life situations, where conditions are usually not steady or uniform.

In addition to theoretical and numerical studies, this research introduces an experimental component: a physical model of channel suspension-carrying flows created at the Institute of Irrigation and Water Problems. This model enables the study of sediment particle distribution by depth, which is critical for understanding suspension and transport processes in natural channels. Traditional modeling approaches, which rely on diffusion theory and assume the equivalence of turbulent momentum and concentration coefficients, are extended in this study by incorporating energy balance equations [23, 24]. While this expansion brings more unknowns, it helps us better understand energy loss and what causes sediment suspension. Adding energy balance gives a new view of how energy impacts the flow's ability to suspend and move particles.

This paper seeks to develop a comprehensive model of channel suspension-carrying flows that fully incorporates interphase forces, turbulence effects, and energy balance considerations. The goal of this research is to improve our understanding of the dynamics of sediment transport by combining insights from previous numerical, theoretical, and experimental studies. This will help us make better predictions for flows of sediment in channel systems.

The following sections present the theoretical background of this model, discuss the effects of transient and non-uniform flow conditions on particle dynamics, and examine how interfacial forces contribute to the overall energy distribution in the system. We aim to bridge the gap between simplified theoretical models and the complex behavior observed in natural and engineered systems, facilitating more accurate predictions of sediment transport and associated hydrodynamic processes.

2. Research methodology

To address the challenging problems associated with suspension-carrying channel flows, this study combines experimental and computational methods to capture the dynamics of sediment-laden flows. Previous studies ([2]; [18]) have highlighted the important role of interfacial forces such as drag, buoyancy, and turbulence modulation in sediment transport. However, while numerical models have improved our understanding, practical applications still need more experimental testing ([4]; [3]). The aim of this study is to bridge this gap by developing a physical model to observe sediment behavior. This is complemented by a strong

numerical framework that includes two-phase flow dynamics and energy balance equations, as suggested by recent studies ([19]; [17]). The methodology described here combines empirical data with state-of-the-art computational simulations to offer an integrated approach to understanding the factors that control sediment suspension and transport.

Theoretical Foundations and Basic Equations

In hydraulics, the Bernoulli equation serves as a fundamental principle for analyzing single-phase fluid flow through pipes, channels, and rivers. This equation helps fluid systems save energy by connecting pressure, velocity, and height in a flow. However, we frequently work with mixtures in real-world situations, such as liquids with gas or vapor bubbles or suspensions or gas suspensions. The single-phase flow assumptions are invalid in these situations. We must modify the fundamental Bernoulli equation to account for extra forces and phase-to-phase interactions due to the complexity of these mixtures.

Modified Bernoulli equations or other momentum conservation equations are applied to such systems. According to studies, the first integrals of the equation of motion can be obtained for both constant and fluctuating concentrations when working with ideal mixes, such as tiny solid particles in a fluid. These integrals provide essential insights for systems with dispersed particles, as demonstrated in works by [25] and [24], where the governing equations for dispersed flow mixtures are expanded to account for multi-phase interactions and the dynamics of energy distribution.

In works on the motion of a dispersed mixture (using the example of a multiphase mixture in the interpenetrating and interacting model of Kh.A. Rakhmatullin [26]). The equation of motion and continuity, written in the Gromeko-Lamb form [23–25]:

$$\begin{aligned} \rho_n \frac{\partial \vec{V}_n}{\partial t} + \rho_n \text{grad} \frac{V_n^2}{2} + 2\rho_n [\vec{\omega}_n, \vec{V}_n] \\ = f_n \mu_n \nabla^2 \vec{V}_n + \rho_n \vec{F}_n - f_n \text{grad} p \\ + K^* (\vec{V}_p - \vec{V}_n) \end{aligned} \quad (1)$$

Continuity equation:

$$\frac{\partial \rho_n}{\partial t} + \text{div}(\rho_n \vec{V}_n) = 0, \quad \sum_{n=1}^N f_n = 1$$

where $\vec{V}_n$ is the velocity vector of the n th phase of the mixture; ρ_n is the reduced density of the n th phase of the mixture, determined by the equality: $\rho_n = f_n \cdot \rho_{ni}$ is the true density of the n th phase of the mixture, μ_n is the dynamic viscosity, $\vec{F}_n$ is the external force per unit mass and $K^* = \sum_{i,j=1}^{N-1} K_{ij}$ is the interaction coefficient of the i and j phases of the mixture.

In the following we will consider a two-phase or three-phase mixture:

in a two-phase flow (flow of water with sediments or salt, etc.) $\vec{K}_1 = K(\vec{V}_p - \vec{V}_n)$

for three-phase flow:

$$\vec{K}_1 = K_{12}(\vec{V}_2 - \vec{V}_1) + K_{23}(\vec{V}_3 - \vec{V}_2) + K_{13}(\vec{V}_1 - \vec{V}_3)$$

Let us assume that external forces are conservative:

$$\vec{F}_n = -\text{grad} U_n$$

the process of movement of the mixture is barotropic (pressure is a function of true densities) $p = p(\rho_{ni})$. Assuming that the interaction coefficients K^* are constant.

Physical Model Setup

In addition to these studies, a physical model of a channel suspension-carrying flow was created in the



laboratory of the Institute of Irrigation and Water Problems. The study of the distribution of suspended sediment particles by the depth of the flow is of great theoretical and practical importance for studying the processes of suspension, attraction and transportation of river sediments. The calculation formulas used in practice are derived from the diffusion theory, based on the assumption of the equality of the coefficient of turbulent movement of momentum and concentration. For the case of steady and uniform movement, an equation was obtained in the form [23, 24] The physical model of a channel suspension-carrying flow adopted by us was formulated and substantiated, characterized by the following main provisions:

1) the flow is the movement of a dispersed medium with a variable mass of its constituent components;

2) the variability of the mass of the components is taken into account by introducing into consideration a system of sources - sinks, which are located according to the law of the function of a simple layer, taking non-zero values on the surface of mass exchange;

3) the discrete structure of the flow consisting of the carrier liquid and suspended particles is taken into account using the Frank discontinuous function s , which takes the values $s = 1$ when a suspended particle is located at the considered point in space, and $s = 0$ when the liquid is at this point;

4) the discrete structure of the suspension-carrying flow in its averaged equations can be reflected by statistical averaging of the flow characteristics (component velocities, pressures, etc.);

5) the concentration $\bar{s}$ is the probability of finding suspended particles at the considered point in space occupied by the suspension-carrying flow. Based on the formulated flow model, a system of equations for the continuity and motion of its components was compiled [23].

Numerical Model and Governing Equations

In the present work, this system is supplemented by energy balance equations. The expansion of the system by energy equations does not lead to a decrease in the degree of its non-closure, but on the contrary, introduces additional unknown quantities. Nevertheless, the issues of energy balance and its dissipation in suspended matter flows are of independent interest, since energy consideration allows us to illuminate the processes of suspension and transfer of suspended matter by a water flow.

The balance equation for the total energy of the flow in the volume τ has the form (2):

$$\frac{d}{dt} \int_{\tau} \rho \left(W + \frac{\vartheta^2}{2} \right) \delta\tau = \int_{\tau} \rho F \vartheta \delta\tau + \int_{\sigma} p_n \vartheta \delta\sigma + \theta \quad (2)$$

where ϑ is the flow velocity vector; p_n is the stress vector on the surface σ of the volume τ ; F is the vector of volume forces; ρ is the density; W is the internal energy (mainly thermal) of a unit of flow volume; θ is the energy influx to the volume τ per unit of time t .

After simple transformations similar to those used in (1), we reduce equation (1) to the following form:

$$\left[\rho_w(1-s) \frac{d}{dt} \left(W + \frac{\vartheta_w^2}{2} \right) + \left(W + \frac{\vartheta_w^2}{2} \right) m_w - \rho_w(1-s) \vartheta_w F - (1-s) \operatorname{div} P_w \vartheta_w - \rho_w(1-s) \theta_w \right] =$$

$$= - \left[\rho_w s \frac{d}{dt} \left(W_s + \frac{\vartheta_s^2}{2} \right) + \left(W_s + \frac{\vartheta_s^2}{2} \right) m_s - \rho_s s \vartheta_s F - \rho_s s \vartheta_s F - s \operatorname{div} P_s \vartheta_s - \rho_s s \theta_s \right]$$

where the indices s and w indicate whether the parameter belongs to the solid or liquid phase of the flow, respectively; P is the stress tensor; θ is the per-second influx of thermal energy to a unit volume of the corresponding phase of the flow; m is the mass of the corresponding phase added to a unit volume of the flow per unit time.

By equating each part of this dependence of the power of interphase interaction, equal to the product of the force R of interaction of the phases in their relative motion and the value of the relative velocity $\vartheta_w - \vartheta_s$, we obtain the energy balance equations for the solid and liquid components of the flow, respectively:

$$\begin{aligned} & \left[\frac{d}{dt} \left(W + \frac{\vartheta_w^2}{2} \right) - \vartheta_w F - \frac{1}{\rho_s} \operatorname{div} P_s \vartheta_s - \theta_s \right] + \left(W_s + \frac{\vartheta_s^2}{2} \right) m_s \\ & = R(\vartheta_w - \vartheta_s) \quad (3) \\ & \left[\rho_w(1-s) \frac{d}{dt} \left(W_w + \frac{\vartheta_w^2}{2} \right) - \vartheta_w F - \frac{1}{\rho_w} \operatorname{div} P_w \vartheta_w - \theta_w \right] + \left(W_w + \frac{\vartheta_w^2}{2} \right) m_w \\ & = -R(\vartheta_w - \vartheta_s) \quad (3') \end{aligned}$$

The change in the internal energy of the volume occurs due to the influx of energy from the outside, as well as due to the dissipation of energy $N_{\text{дис}}$ due to the work of internal forces. Thus, for a solid component, the change in internal energy can be expressed by the equation

$$\frac{d}{dt} \int_{\tau} \rho_s s W_s \delta\tau = \int_{\tau} \rho_s s \theta_s \delta\tau + \int_{\tau} s N_{\text{дис}} \delta\tau$$

from where, bearing in mind that the left-hand side in accordance with the continuity equation (1)

$$\frac{d}{dt} (\rho_s s \delta\tau) = m_s \delta\tau$$

can be represented as:

$$\frac{d}{dt} \int_{\tau} \rho_s s W_s \delta\tau = \int_{\tau} \left[\rho_s s \frac{dW_s}{dt} + W_s m_s \right] \delta\tau$$

we obtain the following equation, which connects the change in the internal energy of the suspension with the dissipation of energy and heat exchange with the environment:

$$\rho_s \frac{dW_s}{dt} + W_s m_s = s \rho_s \theta_s + s N_{\text{дис}}$$

Similarly for liquid:

$$\begin{aligned} & \rho_s(1-s) \frac{dW_s}{dt} + W_s m_s \\ & = (1-s) \rho_s \theta_s + (1-s) N_{\text{дис}} \quad (3'') \end{aligned}$$

Comparing equations (2) and (3) we obtain

$$s \rho_s \left[\frac{d}{dt} \frac{\vartheta_s^2}{2} - \vartheta_w F - \frac{1}{\rho_s} \operatorname{div} P_s \vartheta_s \right] + \frac{\vartheta_s^2}{2} m_s + s N_{\text{дис}} = R(\vartheta_w - \vartheta_s) \quad (4)$$

$$\begin{aligned} & (1-s) \rho_s \left[\frac{d}{dt} \frac{\vartheta_s^2}{2} - \vartheta_w F - \frac{1}{\rho_s} \operatorname{div} P_s \vartheta_s \right] + \frac{\vartheta_s^2}{2} m_s \\ & + (1-s) N_{\text{дис}} = R(\vartheta_w - \vartheta_s) \quad (4') \end{aligned}$$

Or using the continuity equations (1) in the form:



$$\frac{\partial \rho_s s}{\partial t} + \text{div} \rho_s s \vartheta_s = m_s$$

$$\frac{\partial \rho_s (1-s)}{\partial t} + \text{div} [\rho_w (1-s) \vartheta_w] = m_w$$

We have the following energy equations that relate the instantaneous characteristics of the flow:

$$\begin{aligned} \frac{\partial}{\partial t} \left(\rho_s s \frac{\vartheta_s^2}{2} \right) + \text{div} \left(\rho_s s \frac{\vartheta_s^2}{2} \vartheta_s \right) - \rho_s s \vartheta_s F - s \text{div} P_s \vartheta_s \\ + s N_{\text{дис}} \\ = R(\vartheta_w - \vartheta_s) \end{aligned} \quad (5)$$

$$\begin{aligned} \frac{\partial}{\partial t} \left(\rho_s (1-s) \frac{\vartheta_s^2}{2} \right) + \text{div} \left(\rho_s (1-s) \frac{\vartheta_s^2}{2} \vartheta_s \right) \\ - \rho_s (1-s) \vartheta_s F - \\ - (1-s) \text{div} P_s \vartheta_s + (1-s) N_{\text{дис}} \\ = -R(\vartheta_w - \vartheta_s) \end{aligned} \quad (5')$$

After averaging equations (5) over probability [23] with $\rho_s = \text{const}$ and $\rho_w = \text{const}$ we obtain, neglecting the moments of the third order correlation.

$$\begin{aligned} \frac{\partial}{\partial t} \left(\rho_s \bar{s} \frac{\bar{\vartheta}_s^2}{2} \right) + \text{div} \left(\rho_s \bar{s} \frac{\bar{\vartheta}_s^2}{2} \vartheta_s \right) - \rho_s \bar{s} \vartheta_s F - \bar{s} \text{div} \bar{P}_s \bar{\vartheta}_s \\ + \bar{s} \bar{N}_{\text{дис}} - \bar{R}(\bar{\vartheta}_w - \bar{\vartheta}_s) = \\ = -\frac{\partial}{\partial t} \left(\rho_s \bar{s} \frac{\bar{\vartheta}_s'^2}{2} \right) - 3 \text{div} \left(\rho_s \bar{s} \frac{\bar{\vartheta}_s'^2}{2} \vartheta_s \right) + \bar{s} \text{div} \bar{P}_s' \bar{\vartheta}_s' \\ + \bar{R}'(\bar{\vartheta}_w' - \bar{\vartheta}_s') \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{\partial}{\partial t} \left(\rho_w (1-\bar{s}) \frac{\bar{\vartheta}_w^2}{2} \right) + \text{div} \left(\rho_s \bar{s} \frac{\bar{\vartheta}_w^2}{2} \vartheta_w \right) - \rho_s (1-\bar{s}) \bar{\vartheta}_w F \\ - (1-\bar{s}) \text{div} \bar{P}_s \bar{\vartheta}_s + (1-\bar{s}) \bar{N}_{\text{дис}} \\ - \bar{R}(\bar{\vartheta}_w - \bar{\vartheta}_s) = \\ = -\frac{\partial}{\partial t} \left(\rho_s (1-\bar{s}) \frac{\bar{\vartheta}_s'^2}{2} \right) - 3 \text{div} \left(\rho_s (1-\bar{s}) \frac{\bar{\vartheta}_s'^2}{2} \vartheta_s \right) \\ + (1-\bar{s}) \text{div} \bar{P}_s' \bar{\vartheta}_s' + \bar{R}'(\bar{\vartheta}_w' - \bar{\vartheta}_s') \end{aligned} \quad (6')$$

Energy equations (5) and (6) are equally valid when considering suspended flows both with and without sources. These equations show that the energy balance of the flow components consists of both kinetic energy and energy caused by mass forces, pressure forces, dissipative forces, and interphase interaction forces.

To obtain an expression characterizing the energy dissipated in a suspension-carrying flow

$$N_{\text{дис}} = (1-\bar{s}) \bar{N}_{\text{дис}_w} + \bar{s} N_{\text{дис}_s}$$

using the equations of motion of the components (1), which after scalar multiplication by $\bar{\vartheta}_s$ and $\bar{\vartheta}_w$ respectively and some transformations can be represented in the following form:

$$\begin{aligned} \frac{\partial}{\partial t} \left(\rho_s \bar{s} \frac{\bar{\vartheta}_s^2}{2} \right) + \text{div} \left(\rho_s \bar{s} \frac{\bar{\vartheta}_s^2}{2} \vartheta_s \right) - \rho_s \bar{s} \vartheta_s F - \bar{s} \text{div} \bar{P}_s \\ + \frac{\bar{\vartheta}_s^2}{2} m_s + \rho_s \bar{\vartheta}_s \text{Div} [\bar{s} \vartheta_s' \vartheta_s'] \\ = \bar{\vartheta}_s \bar{R} \end{aligned} \quad (7)$$

$$\begin{aligned} \frac{\partial}{\partial t} \left(\rho_w (1-\bar{s}) \frac{\bar{\vartheta}_w^2}{2} \right) + \text{div} \left(\rho_s (1-\bar{s}) \frac{\bar{\vartheta}_w^2}{2} \vartheta_w \right) \\ - \rho_w (1-\bar{s}) \bar{\vartheta}_w F - (1-\bar{s}) \bar{\vartheta}_w \text{Div} \bar{P}_w \\ + \end{aligned}$$

$$\begin{aligned} + \frac{\bar{\vartheta}_w^2}{2} m_w + \rho_w \bar{\vartheta}_w \text{Div} [(1-\bar{s}) \vartheta_w' \vartheta_w'] \\ = -\bar{\vartheta}_w \bar{R} \end{aligned} \quad (7')$$

From a comparison of equations (6) and (7), bearing in mind [27] that

$$\bar{\vartheta} \text{Div} \bar{P} - \bar{\vartheta} \text{Div} \bar{P} \bar{\vartheta} = -(\bar{P} \nabla) \bar{\vartheta} = -\bar{P} \bar{\Delta}$$

where $\bar{\Delta}$ is the strain rate tensor, we obtain the following expressions:

$$\begin{aligned} \bar{s} \bar{N}_{\text{дис}} - \bar{s} \bar{P}_s \bar{\Delta}_s - \frac{\bar{\vartheta}_s^2}{2} m_s - \bar{R}(\bar{\vartheta}_w - 2\bar{\vartheta}_s) \\ = -\frac{\partial}{\partial t} \left[\rho_s \bar{s} \frac{\bar{\vartheta}_s'^2}{2} \right] - 3 \text{div} \left[\rho_s \bar{s} \frac{\bar{\vartheta}_s'^2}{2} \vartheta_s \right] \\ + \\ + \bar{s} \text{div} \bar{P}_s' \bar{\vartheta}_s' + \bar{\vartheta}_s \text{Div} [\rho_s \bar{s} \bar{\vartheta}_s' \vartheta_s'] + \bar{R}(\bar{\vartheta}_w' - \bar{\vartheta}_s') \\ (1-\bar{s}) \bar{N}_{\text{дис}_w} - (1-\bar{s}) \bar{P}_w \bar{\Delta}_w - \frac{\bar{\vartheta}_w^2}{2} m_w - \bar{R} \bar{\vartheta}_s \\ = -\frac{\partial}{\partial t} \left[\rho_w (1-\bar{s}) \frac{\bar{\vartheta}_w'^2}{2} \right] \\ - 3 \text{div} \left[\rho_s (1-\bar{s}) \frac{\bar{\vartheta}_s'^2}{2} \vartheta_s \right] + \\ + (1-\bar{s}) \text{div} \bar{P}_s' \bar{\vartheta}_s' + \bar{\vartheta}_w \text{Div} [\rho_s (1-\bar{s}) \bar{\vartheta}_s' \vartheta_s'] + \bar{R}(\bar{\vartheta}_w' - \bar{\vartheta}_s') \end{aligned}$$

Summing up the obtained dependencies gives an expression for the energy dissipated in the suspension-carrying flow:

$$\begin{aligned} \bar{N}_{\text{дис}_w} = (1-\bar{s}) \bar{P}_w \bar{\Delta}_w + \bar{s} \bar{P}_s \bar{\Delta}_s + \frac{\bar{\vartheta}_w^2}{2} m_w + \frac{\bar{\vartheta}_s^2}{2} m_s \\ + \bar{R}(\bar{\vartheta}_w - \bar{\vartheta}_s) - \\ - \frac{\partial}{\partial t} \left[\rho_w (1-\bar{s}) \frac{\bar{\vartheta}_w'^2}{2} + \rho_s \bar{s} \frac{\bar{\vartheta}_s'^2}{2} \right] \\ - 3 \text{div} \left[\rho_s (1-\bar{s}) \frac{\bar{\vartheta}_s'^2}{2} \vartheta_s + \rho_s \bar{s} \frac{\bar{\vartheta}_s'^2}{2} \vartheta_s \right] \\ + \\ + (1-\bar{s}) \text{div} \bar{P}_w' \bar{\vartheta}_w' + \bar{s} \text{div} \bar{P}_s' \bar{\vartheta}_s' + \bar{\vartheta}_w \text{Div} [\rho_s (1-\bar{s}) \bar{\vartheta}_s' \vartheta_s'] \\ + \bar{\vartheta}_s \text{Div} [\rho_s \bar{\vartheta}_s' \vartheta_s'] \end{aligned} \quad (8)$$

Let us introduce the following notations into equation (8).

$$\begin{aligned} I = \bar{N}_{\text{дис}_w} &= (1-\bar{s}) \bar{P}_w \bar{\Delta}_w + \bar{s} \bar{P}_s \bar{\Delta}_s, \\ II &= \frac{\bar{\vartheta}_w^2}{2} m_w + \frac{\bar{\vartheta}_s^2}{2} m_s, \\ III &= \bar{R}(\bar{\vartheta}_w - \bar{\vartheta}_s) \\ IV &= -\frac{\partial}{\partial t} \left[\rho_w (1-\bar{s}) \frac{\bar{\vartheta}_w'^2}{2} + \rho_s \bar{s} \frac{\bar{\vartheta}_s'^2}{2} \right] \\ &\quad - 3 \text{div} \left[\rho_s (1-\bar{s}) \frac{\bar{\vartheta}_s'^2}{2} \vartheta_s + \rho_s \bar{s} \frac{\bar{\vartheta}_s'^2}{2} \vartheta_s \right] \\ &\quad + \\ &\quad + (1-\bar{s}) \text{div} \bar{P}_w' \bar{\vartheta}_w' + \bar{s} \text{div} \bar{P}_s' \bar{\vartheta}_s' + \bar{\vartheta}_w \text{Div} [\rho_s (1-\bar{s}) \bar{\vartheta}_s' \vartheta_s'] \\ &\quad + \bar{\vartheta}_s \text{Div} [\rho_s \bar{\vartheta}_s' \vartheta_s'] \end{aligned}$$

Thus, energy dissipation in a channel suspension-carrying flow occurs due to the work of internal forces (I), the addition of additional mass (II), the interaction of flow components (III), and turbulent energy conductivity (IV).

In flows of pure liquid ($\bar{s} = 0$) energy dissipation is determined by the expression



$$\begin{aligned} \bar{N}_{\text{дис}w}^0 &= \bar{P}_w \bar{A}_w + \frac{\bar{\vartheta}_w^2}{2} m_w - \frac{\partial}{\partial t} \left[\rho_w \frac{\bar{\vartheta}_w'^2}{2} \right] - 3 \text{div} \left[\rho_w \frac{\bar{\vartheta}_s'^2}{2} \right] \\ &+ \text{div} \bar{P}_w' \bar{\vartheta}_w' + \bar{\vartheta}_w \text{Div} [\rho_s \bar{\vartheta}_w' \bar{\vartheta}_s'] \end{aligned} \quad (9)$$

It can be assumed that the difference in energy dissipation in the suspended-carrying flow and in the flow of pure liquid is nothing other than the energy $\bar{N}_{\text{B3B}}$ expended on involving the suspension in movement and transport ("weighing work"). However, bearing in mind that the intensity of turbulent pulsations in the flow of pure liquid and the liquid transporting the suspension are different from each other at the same values of the averaged characteristics of the flow, we introduce into consideration the parameter,

$$A = \int_0^s -\frac{\partial}{\partial s} \left[-\frac{\partial}{\partial t} \rho_w \frac{\bar{\vartheta}_w'^2}{2} \right] - 3 \text{div} \left[\rho_w \frac{\bar{\vartheta}_w'^2}{2} \right] + \text{div} \bar{P}_w' \bar{\vartheta}_w' + \bar{\vartheta}_w \text{Div} [\rho_s \bar{\vartheta}_w' \bar{\vartheta}_s'] ds$$

taking into account the change in the intensity of turbulent pulsations in the liquid flow in the presence of suspended particles. Thus,

$$\begin{aligned} \bar{N}_{\text{B3B}w} &= R(\bar{\vartheta}_w - \bar{\vartheta}_s) - \bar{s} \bar{P}_w \bar{A}_w + \bar{s} \bar{P}_s \bar{A}_s + \frac{\bar{\vartheta}_s^2}{2} m \\ &+ \frac{\partial}{\partial t} \left[\rho_w \bar{s} \frac{\bar{\vartheta}_w'^2}{2} - \rho_s \bar{s} \frac{\bar{\vartheta}_s'^2}{2} \right] + \\ &+ 3 \text{div} \left[\bar{s} \rho_w \frac{\bar{\vartheta}_s'^2}{2} \bar{\vartheta}_s + \rho_s \bar{s} \frac{\bar{\vartheta}_s'^2}{2} \bar{\vartheta}_s \right] - \bar{s} \text{div} (\bar{P}_w' \bar{\vartheta}_w' - \bar{P}_s' \bar{\vartheta}_s') \\ &- \bar{\vartheta}_w \text{Div} [\rho_w \bar{\vartheta}_w' \bar{\vartheta}_w'] + \bar{\vartheta}_s \text{Div} [\rho_s \bar{s} \bar{\vartheta}_s' \bar{\vartheta}_s'] \\ &+ A \end{aligned}$$

Assuming further that $P_w \approx P_s \approx pE$, where p is the hydrodynamic pressure, E is the unit tensor, we have $P \bar{A} = -p \text{div} \bar{\vartheta}$, from which we obtain the following expression for the weighing work:

$$\begin{aligned} \bar{N}_{\text{B3B}w} &= R(\bar{\vartheta}_w - \bar{\vartheta}_s) + \bar{s} p \text{div} (\bar{\vartheta}_w - \bar{\vartheta}_s) + \frac{\bar{\vartheta}_s^2}{2} m_s \\ &+ \frac{\partial}{\partial t} \left[\rho_w \bar{s} \frac{\bar{\vartheta}_w'^2}{2} - \rho_s \bar{s} \frac{\bar{\vartheta}_s'^2}{2} \right] + \\ &+ 3 \text{div} \left[\bar{s} \rho_w \frac{\bar{\vartheta}_s'^2}{2} \bar{\vartheta}_s - \rho_s \bar{s} \frac{\bar{\vartheta}_s'^2}{2} \bar{\vartheta}_s \right] - \bar{s} \text{div} p' (\bar{\vartheta}_w' - \bar{\vartheta}_s') \\ &- \bar{\vartheta}_w \text{Div} [\rho_w \bar{\vartheta}_w' \bar{\vartheta}_w'] + \bar{\vartheta}_s \text{Div} [\rho_s \bar{s} \bar{\vartheta}_s' \bar{\vartheta}_s'] \\ &+ A \end{aligned} \quad (10)$$

The obtained expression for the weighing work can be represented as the sum of the weighing works of the averaged and pulsating motion:

$$\begin{aligned} \bar{N}_{\text{B3B}}^{\text{опр}} &= R(\bar{\vartheta}_w - \bar{\vartheta}_s) \bar{s} p \text{div} (\bar{\vartheta}_w - \bar{\vartheta}_s) + \frac{\bar{\vartheta}_s^2}{2} m_s, \\ \bar{N}_{\text{B3B}}^{\text{пульс}} &= \frac{\partial}{\partial t} \left[\rho_w \bar{s} \frac{\bar{\vartheta}_w'^2}{2} - \rho_s \bar{s} \frac{\bar{\vartheta}_s'^2}{2} \right] \\ &+ 3 \text{div} \left[\bar{s} \rho_w \frac{\bar{\vartheta}_s'^2}{2} \bar{\vartheta}_s - \rho_s \bar{s} \frac{\bar{\vartheta}_s'^2}{2} \bar{\vartheta}_s \right] + \\ &+ \bar{s} \text{div} p' (\bar{\vartheta}_w' - \bar{\vartheta}_s') - \bar{\vartheta}_w \text{Div} [\rho_w \bar{\vartheta}_w' \bar{\vartheta}_w'] + \bar{\vartheta}_s \text{Div} [\rho_s \bar{s} \bar{\vartheta}_s' \bar{\vartheta}_s'] \\ &+ A \end{aligned} \quad (11)$$

Under conditions of a steady longitudinally uniform plane suspension-carrying flow and under the assumption that the vertical component of the relative velocity ($\bar{\vartheta}_w - \bar{\vartheta}_s$) of the hydraulic size w of the sediments is equal and that the horizontal component ($\bar{\vartheta}_w - \bar{\vartheta}_s$) is equal to zero, i.e., under the conditions considered by M.A. Velikanov [25] when substantiating the gravitational theory,

expression (10) for the weighing work is significantly simplified:

$$\begin{aligned} \bar{N}_{\text{B3B}w} &= \bar{s} g (\rho_s - \rho_w) w + 3 \rho_s w \frac{\partial}{\partial y} \left[\bar{s} \frac{\bar{\vartheta}_w'^2}{2} \right] - \\ &- 3 (\rho_s - \rho_w) \frac{\partial}{\partial y} \left[\bar{s} \frac{\bar{\vartheta}_w'^2}{2} \bar{\vartheta}_w \right] - \rho_s w \frac{\partial}{\partial y} [\bar{s} \bar{\vartheta}_s' \bar{\vartheta}_s'] + A \end{aligned}$$

and in contrast to the formula $\bar{N}_{\text{B3B}w} = \bar{s} g (\rho_s - \rho_w) w$ used by M.A. Velikanov, takes into account the dissipation of pulsating energy.

3. Results and discussion

The analysis developed in the preceding section yields a description of how mechanical energy is stored, exchanged, and dissipated in a channel flow that transports suspended sediment. Instead of a single friction law, it produces a coupled set of energy-balance equations for the solid and liquid phases — equations (5)–(7) for the instantaneous and averaged motion and equations (8)–(11) for the dissipated and weighing work — from which the mechanisms responsible for suspension can be read directly.

Mechanisms of energy dissipation. Equation (8), together with the grouping introduced through the terms I–IV, shows that dissipation in a suspension-carrying flow is not controlled by viscous friction alone. Four physically distinct contributions appear. Term I is the work of the internal pressure and viscous stresses in each phase, weighted by its volumetric share; this is the only mechanism retained in the classical single-phase description. Term II, proportional to the added masses m_w and m_s , is the energy tied to the exchange of mass between the phases through the system of sources and sinks, and it vanishes whenever the mass of the components is treated as constant. Term III, the product of the interphase force R and the relative velocity of the phases, is the work done in the relative slip between water and particles — the drag-type coupling stressed in two-fluid models of sediment transport. Term IV collects the turbulent, or pulsation, transport of energy together with its rate of change. The presence of terms II and III alongside the familiar I and IV is the central structural result: in a real sediment-laden channel, energy is lost not only to viscous friction and turbulence, but also to sustaining mass exchange and to the relative motion between the phases.

Consistency in the clear-water limit. When the concentration vanishes, equation (8) reduces to equation (9), the dissipation of a turbulent flow of pure liquid. This limit is both an internal check on the derivation and a means of isolating the extra energy demanded by the particles: the difference between the dissipation in the sediment-laden flow and that in clear water is interpreted as the energy spent in holding the sediment in suspension and moving it downstream — the weighing work.

Energetics of suspension. Because the intensity of the turbulent pulsations in clear water is not the same as in the identical flow carrying particles, this difference cannot be formed by simple subtraction. The correction is carried by the parameter A , which integrates the change in pulsation energy with concentration. With this correction the weighing work is given by equation (10) and, once an isotropic stress state is assumed, it separates into an averaged-motion part and a pulsating part, equation (11). This separation carries the main physical message of the work: suspension is maintained partly by the mean relative motion and the



interphase force, and partly by the turbulent pulsations themselves. The finding places the role of turbulence, identified as the controlling factor in earlier studies of sediment suspension (Huang et al. 2019; Jha and Bombardelli 2011), inside an explicit energy budget rather than a diffusion analogy.

Relation to the gravitational theory. Under the conditions of Velikanov's gravitational theory — steady, longitudinally uniform plane flow in which the vertical relative velocity equals the hydraulic size w of the particles and the horizontal relative velocity is zero — the general expression (10) collapses to the reduced form obtained at the close of the derivation. Its leading term is the product of the concentration, the submerged specific weight $g(\rho_s - \rho_w)$, and the hydraulic size w , which is exactly the weighing work of the gravitational theory; that classical result therefore appears here as a special case. The remaining terms, built from gradients of the pulsation energy, are the corrections the present formulation adds, and they represent the dissipation of pulsating energy that the gravitational theory leaves out. This offers a resolution to the long-standing objection that a purely gravitational accounting misstates the energy required for suspension, by locating the missing part in the turbulent pulsation field.

Comparison with earlier energy formulations. The energy-balance equations obtained here depart from those of Barenblatt and of Frankl in three ways that follow directly from the model assumptions. The variability of the mass of the flow components is retained through the source-sink system (term II); the change in the intensity of the liquid's turbulent pulsations caused by the transported particles is included through the parameter A ; and the work of the interphase interaction forces is represented more completely, in both the averaged and the pulsating motion (term III). Each of these generalises the earlier steady, constant-mass descriptions toward the unsteady, non-uniform conditions that characterise natural channels.

Limitations and outlook. Adding the energy equations to the system does not close it; as noted in the derivation, each new balance introduces further unknown correlations. The value of the formulation is therefore diagnostic rather than immediately predictive: it identifies the quantities that must be measured or modelled — the interphase force R , the added-mass terms, and the particle-modified pulsation field — before the energy budget can be evaluated for a given flow. This is where the physical model of the channel suspension-carrying flow is intended to contribute, by supplying the depth distribution of concentration and the pulsation data needed to calibrate these terms and to compare the magnitudes of the averaged and pulsating weighing work under controlled conditions.

4. Conclusion

This research contributes to the broader understanding of sediment transport. It highlights the role of energy dissipation and interfacial interactions. These diffusion-based factors are often underestimated in traditional models. The energy balance equations for the suspended matter-carrying flow obtained in this work differ from similar equations compiled earlier by G. I. Barenblatt [24, 28, 29] and F. I. Frankl [30, 31] by taking into account the variability of the mass of the flow components, by taking into account the change in the intensity of turbulent pulsations of the liquid when it transports suspended matter, and by more

fully taking into account the work of the forces of interphase interaction.

Overall, the improved modeling framework established here offers a robust approach to studying sediment-laden flows, providing new insights that can inform both theoretical studies and practical decisions in fluid mechanics and environmental hydraulics.

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