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Mathematical modeling of transient groundwater filtration in multilayered media with a low-permeability barrier

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Abstract: The article numerical models and computational algorithms for pressure and pressureless filtration processes in single-layer porous media were analyzed, and numerical models for pressure-pressureless filtration problems in dynamically coupled, low-permeable double-layer porous media based on the laws of fluid motion were developed, and software was created to solve groundwater filtration problems based on mathematical models and computational algorithms in the Matlab software tool.
The model and software they create are used to predict groundwater movement, assess environmental safety, or manage water resources.

Keywords: Filtration processes, mathematical models, groundwater filtration issues, software interface

1. Introduction

In our republic, research on filtration theory is of particular interest for accurately predicting groundwater level changes during irrigation, as well as assessing the impact of artificial and natural drainage structures on groundwater fluctuations. These studies are also of great importance for hydrogeology, land reclamation, and soil science.

Numerous hydrogeological cross-section studies have shown that, in most cases, the main aquifer, from which water is extracted, is overlain by a low-permeability cover layer and confined at the bottom by a weakly permeable barrier. This barrier facilitates the connection with underlying aquifers.

The problem of hydrogeological calculations for aquifers, oil, and gas fields remains unresolved. The mathematical challenges encountered in this field have forced researchers to simplify and schematize the physical picture of water movement in stratified conditions. However, the demand for scientifically grounded filtration theory predictions continues to grow.

The selection of optimal mathematical models for the studied process is impossible without a thorough quantitative analysis of various natural and artificial factors influencing the process. This analysis is best conducted through computational experiments using electronic computers. The process consists of several stages: problem formulation, mathematical modeling, computational algorithm development, programming, result analysis, and verification of the model's adequacy.

One of the most important aspects of comprehensive research is mathematical modeling of aquifers using various filtration theory models. The most effective and efficient methods for solving filtration theory problems involve numerical and computer modeling.

Since it is impossible to study water filtration processes in deep underground layers under natural conditions, mathematical, numerical, and computer modeling methods are employed. Through modeling, we can study, analyze, and predict filtration processes and groundwater movement in deep layers of the Earth.

The primary method of cultivating agricultural crops in Central Asia is artificial irrigation. The development of irrigated agriculture is constrained by water resources, making the extensive use of groundwater—an internal reserve—particularly important. The construction of hydraulic and reclamation structures, along with the utilization of groundwater for water supply and irrigation, affects the balance and regime of groundwater. These changes can lead either to the depletion of reserves or to groundwater rise, resulting in soil salinization. Therefore, the dynamic assessment of exploitable groundwater reserves must be based on comprehensive studies that consider climatic conditions, hydraulic engineering, land reclamation, and water supply.

A comprehensive study of hydrodynamic processes occurring in aquifers is becoming increasingly important due to the development of automated groundwater management systems.

The book by Davydov L.K. and others [1], which has long served as a textbook on general hydrology for university students, presents the fundamentals of hydrology, describes the interconnections between the waters of the Earth, and explains the general laws governing hydrological processes in oceans, seas, rivers, groundwater, lakes, reservoirs, swamps, and glaciers.

According to this book, the total amount of water on Earth that is not chemically or physically bound to the Earth's crust and mantle is approximately 1.5 billion km³. About 94% (1.37 billion km³) of this volume is contained in oceans and seas. The volume of water in the atmosphere is relatively small, amounting to about 14,000 km³.

The volume of free gravitational water within the five-kilometer-thick continental crust is estimated at 60 million km³, which is four times greater than the volume of surface water. In the Earth's mantle, there is at least 13–15 billion km³ of chemically bound water—approximately 13 to 15 times the amount found in the world's oceans and on land.

Groundwater, located within the Earth's crust, exists in a zone where water is not the primary component of matter but is instead an integral part of it. The study of groundwater falls under the domain of hydrology—a branch of geology. One of the key properties of water as a liquid is its mobility. The study of the laws governing the motion and equilibrium

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of liquids is the task of hydromechanics and its applied branch—hydraulics. Hydraulics focuses on developing methods to apply general principles of liquid motion and equilibrium to solving practical problems under specific natural and human-made conditions.

This book is particularly useful as it provides a comprehensive list of the thermophysical properties of water in its three aggregate states, as well as details on the Earth's water cycle.

Book [2] is dedicated to the analytical theory of heat and mass transfer phenomena in gas mixtures, dispersed systems, and capillary-porous bodies. The development of mathematical models for filtration processes is based on fundamental physical laws, specifically Fick’s law for mass transfer, Newton’s law for momentum transfer, and Fourier’s law for heat transfer.

Fick’s law assumes a linear relationship between the substance flux and the gradient of the substance concentration within the studied field. However, practical observations show that this law does not always accurately describe filtration processes. To account for anomalous effects, an inertial term—representing the time derivative of the flux density—is sometimes included in Fick’s law. This leads to hyperbolic equations for mass transfer, which possess the property of finite propagation speed for concentration profiles, resulting in the formation of concentration waves.

Macroscopic modeling has demonstrated that the relaxation time of mass flux density is very small. The study [3] shows that, depending on the differentiation of various parameters, different transport laws can be derived, each with distinct characteristics of concentration wave propagation.

Fractional derivatives have been widely used in numerous studies on filtration processes [4, 5, 6, 7]. One such study is the dissertation [8], which focuses on the further development of filtration theory for heterogeneous fluids in various porous media using the mathematical framework of fractional derivatives. It presents a comprehensive analysis of filtration in both homogeneous and heterogeneous media, employing fractional-order derivatives and their computation methods based on Riemann-Liouville, Grünwald-Letnikov, and Caputo definitions.

The key outcome of this research is the development and testing of new modifications of mathematical models for filtration processes. The study provides a qualitative analysis of the results obtained by applying these models to specific problems and highlights variations in modifications that differ from classical results in terms of process intensity over time and spatial distribution.

Study [9] examines the filtration process in a fractured-porous medium, modeled using fractal geometry. Due to the complex structure of fractures and pore blocks, the trajectory of suspended particles follows an intricate path. This study presents one of the first models of mass transport in such media. Within the framework of the bicontinuum approach, convective transport equations containing fractional derivatives are analytically derived for fractured-porous media.

Study [10] investigates hydrodynamic filtration models of heterogeneous fluids in porous media, considering diffusion, hydrodynamic dispersion, convection, adsorption, the heterogeneity of pore volume in terms of filling degree, and internal diffusive mass exchange.

This study provides key information on the adsorption of substances during convective-diffusive transport in porous media. It offers a detailed review of models and methods for solving mass transport problems in porous media. The research addresses mass transport problems in porous media saturated with both stationary and mobile fluids, under various adsorption and internal mass exchange laws (linear, nonlinear, equilibrium, and non-equilibrium). These problems are solved numerically, and concentration profiles of substances in mobile fluid zones, adsorbed substances, and mass exchange between zones are determined for different initial parameter values.

Additionally, the study explores a two-site adsorption model, where the total adsorption surface is divided into two parts—one undergoing equilibrium adsorption and the other non-equilibrium adsorption. The influence of this dual-site adsorption on mass transport characteristics is evaluated. A problem involving mass transport in a porous medium with a wetting front condition is also solved, with differences highlighted between this setting and the semi-infinite reservoir case.

Furthermore, a two-dimensional mass transport problem in a heterogeneous medium is considered, where the fluid remains immobile in certain inclusions but undergoes diffusion transport. The effects of stationary liquid zones and adsorption phenomena on mass transport characteristics in both regions are analyzed. The study also examines salt transport and dissolution processes in porous media and provides solutions for convective-diffusive transport of dissolved salts under wetting front conditions, considering a piecewise-homogeneous initial salinity field.

Maintaining reservoir pressure in oil and gas fields is a pressing issue, as it helps reduce the energy consumption of hydrocarbon extraction. Various injection methods have been proposed [11], including water injection, the disposal of industrial waste containing surface-active agents [12], dry gas injection, in-situ combustion of gas components, high-frequency reservoir stimulation, and other filling techniques.

The dissertation [13] is dedicated to the development of mathematical and numerical modeling methods for filtration processes in single-phase and two-phase media within single-layer and multilayer porous environments.

The review chapter of the study presents key concepts and methods for the development of oil and gas fields under various geological conditions. It also provides an overview of research on modeling unsteady filtration processes of liquids and gases in porous media, as well as an analysis of the main stages of mathematical modeling and computational experiments for oil and gas field development.

Numerical simulations have been conducted to model filtration processes related to the displacement of target products in porous media under water drive conditions. The study discusses the characteristics of field development under water drive mechanisms, considering different types of water flooding: intra-contour and extra-contour flooding in various configurations (well row flooding, areal flooding, selective flooding, localized flooding, frontal flooding, and barrier flooding). The dissertation presents both mathematical and numerical models, along with a computational algorithm for solving two-dimensional filtration problems involving multiphase media. These models account for the moving boundary between oil and water, as well as gas and water. One of the key achievements of this work is the development of a differential finite-difference method that utilizes the advantages of the longitudinal-transverse sweep technique.

The dissertation by U.J. Saidullaev [14] focuses on studying the peculiarities of filtration processes in conditions where a cake layer forms. The main contributions of this work include:

- The enhancement of a mathematical model describing the filtration of heterogeneous liquids with sediment formation on filter surfaces, based on exponential and nonlinear consolidation laws.
- The refinement of a mathematical model for suspension filtration with cake layer formation on filter surfaces, considering convective transport.
- The development of a mathematical model for filtration and suspension filtration with sediment formation on filter surfaces, using a generalized relaxation-based Darcy law.
- The formulation of a system of differential equations for relaxation-based filtration of suspensions with cake layer formation under constant filtrate flow conditions. This system was numerically solved for different relaxation time values.

Cake layer formation occurs naturally. At the same time, various methods and materials are used to suppress the filtration process. In particular, study [15] proposes using bentonite from various deposits in the Republic of Uzbekistan to reduce filtration processes. Based on extensive experimental data, optimal bentonite concentrations and layer thicknesses were determined for earthen dams, considering the physico-mechanical properties of both the soil and bentonite.

Study [16] focuses on problems related to mass transport in aggregated porous media with specific geometries, considering adsorption phenomena and the heterogeneous distribution of filtration velocity fields. The dissertation analyzes challenges in modeling mass transport processes in heterogeneous porous media and investigates the impact of adsorption phenomena on filtration.

A numerical solution has been formulated for the movement of fluids and mass transport in a cylindrical two-layer medium, where the inner cylindrical region represents a macropore, and the outer cylindrical region represents a micropore. The study examines fluid flow and mass transport in a dual-zone cylindrical porous medium, taking into account the non-uniform distribution of filtration velocity fields. Additionally, inverse problems for mass transport have been formulated and solved to determine the mass transfer coefficient, both with and without the consideration of adsorption effects.

2. Research methodology

The mathematical model of the two-dimensional problem of confined-unconfined filtration is reduced to the integration of a quasi-linear system of partial differential equations of the parabolic type:

$$\left.\begin{aligned}
&\frac{\mu_1}{x}\frac{\partial H}{\partial t} = \frac{\partial}{\partial x}\left[(H-b)\frac{\partial H}{\partial x}\right] + \frac{\partial}{\partial y}\left[(H-b)\frac{\partial H}{\partial x}\right] + \frac{f_1}{k_1},\\
&\frac{\mu_2}{k_2}\frac{\partial Z}{\partial t} = \frac{\partial}{\partial x}\left[(Z-b)\frac{\partial Z}{\partial x}\right] + \frac{\partial}{\partial y}\left[(Z-b)\frac{\partial Z}{\partial y}\right] + \frac{f_2}{k_2} - (1 - \frac{h}{Z-b}),\\
&\frac{\mu_3}{T}\frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{k_2}{T}(1 - \frac{h}{Z-b})
\end{aligned}\right\}$$

(1)

with the corresponding boundary conditions:

$$H(x,y,0) = F_1(x,y), (0 \le x, y \le L_1); \quad (2)$$

$$\left.\begin{aligned} Z(x,y,0) = F_2(x,y),\\ h(x,y,0) = F_3(x,y),\end{aligned}\right\}; (0 \le x, y \le L_2); \quad (3)$$

$$k_1(H-b)\frac{\partial H}{\partial x}\bigg|_{x=0} = Q_1(t); \quad (4)$$

$$T\frac{\partial h}{\partial x}\bigg|_{x=L_2} = Q_2(t); \frac{\partial Z}{\partial x}\bigg|_{x=L_2} = 0; \quad (5)$$

$$k_1(H-b)\frac{\partial H}{\partial x} = k_2(Z-b)\frac{\partial Z}{\partial x} + T\frac{\partial h}{\partial x}; x = L_1; \quad (6)$$

Where $H(x,0) = \phi_1(x), 0 \le x \le l_1$; - groundwater level, impermeable layer, filtration coefficients, specific yield, and infiltration in the single-layer filtration zone, respectively.

$Z(x,t), b(x), k_2, \mu_2, f_2$- the same for the covering thickness of the layered filtration zone;

$h(x,t), T, \mu_3$- pressure, coefficients of water conductivity and elastic water yield in the pressure horizon of the layered filtration zone;

Q_1 and Q_2 – lateral inflows into the filtration zone;

L_1- zone separation boundary;

L_2- filtration area length;

$F_1(x), F_2(x), F_3(x)$ - specified functions.

3. Computational experiment

A program was developed in MATLAB for the proposed algorithm. Using this program, the influence of filtration parameters of aquifers on the dynamics of groundwater level and head distribution in the transition zone from a single-layer to a layered filtration zone was studied.

In particular, the influence of the filtration coefficients k_1 and k_2 on changes in levels and pressures was assessed. The remaining geofiltration parameters are considered constant. The obtained forecast calculations for the second and fifth years after the start of irrigation are given in Table 1.

Table 1

The obtained forecast calculations

№	Filtration coefficient, m/day.		Forecast levels at the transition boundary (in meters)	
	Single layer zone k_1	Layered zone k_2	for 2 years	for 5 years
1	5	0,5	431,958	441,026
2	2,5	0,5	427,506	433,689
3	0,5	0,5	421342	422,961
4	0,5	2,5	418,752	418,343

Table 2

The results of the numerical experiment

№	Gravity drainage		Elastic water yield of the layered zone μ^*	Forecast levels at the transition boundary (in meters)	
	Single layer zone μ_1	Layered zone μ_2		for 2 years	for 5 years
1	0,15	0,10	0,08	422,342	423,961
2	0,15	0,10	0,003	419,803	422,417
3	0,15	0,15	0,15	421,318	422,920
4	0,10	0,35	0,08	423,698	425,218

From Table 2 it is evident that the greater the difference in the values of the filtration coefficients of the single-layer

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and layered zones, the greater the height of the rise in the groundwater level at the transition boundary (the initial level is 421.00).

Next, the effect of changing the capacitive properties of μ_1,μ_2 and μ^* on changing the groundwater level at the transition boundary is considered. The remaining geofiltration parameters remain unchanged. The results of the numerical experiment are given in Table 2.

The analysis of the obtained results shows that, as in the previous case, the greater the difference in the capacitive properties of the two zones, the higher the predicted level at the transition boundary. This leads to the accumulation of groundwater reserves in the single-layer unconfined zone.

Currently, we are studying even more complex mathematical models of aquifers, where two highly permeable heterogeneous aquifers, connected by a weakly permeable layer, interact with groundwater flows in the covering thickness through a partition. In this case, infiltration, evaporation, and the operation of drainage structures are taken into account.

Thus, a thorough analysis of various aquifer models and groundwater flow theories has allowed us to effectively apply the developed algorithms and programs for calculating many real-world objects and to provide practical recommendations for improving the reclamation conditions of old irrigated and newly developed areas in arid zones.

The algorithms described in the previous chapters are implemented as universal programs written in the Matlab algorithmic language. Using these programs, various options for predicting problems of pressure and gravity waters for a single-layer model of aquifers are solved, taking into account all natural and artificial factors that affect the filtration process.

The initial information for the programs is prepared as follows. The grid filtration area is supplemented to a rectangle with fictitious nodes. The input procedure must ensure the input of levels (pressures) and filtration coefficients (water conductivity) into two-dimensional arrays with the identifier HN, (TN) (the values of these functions in fictitious nodes are arbitrary).

All collected information about the initial conditions of the filtration area, i.e. about the position of the groundwater level, boundary conditions, design parameters and others constitute an information array that is constantly increasing, expanding and supplemented with more detailed and accurate new data. Therefore, along with the correct collection of information, it is necessary to ensure the most rational forms of storage in the form of technical media convenient for prompt input into a computer.

Development of stable computational schemes, universal and effective algorithms and software that allow for solving problems of pressure and gravity filtration significantly increases the reliability of the resulting numerical calculations and their visualization in the graphical form of the object.

Therefore, in order to improve the efficiency of using modern computers, it is necessary to create software for conducting a computational experiment with interaction with computer specialists. This is necessary for making final or intermediate decisions in the process of analysis and monitoring.

Based on the mathematical model and calculation algorithm on the Matlab software tool, software has been developed for solving groundwater filtration problems. The interface of the software for solving pressure filtration problems is shown in Fig. 3.1.

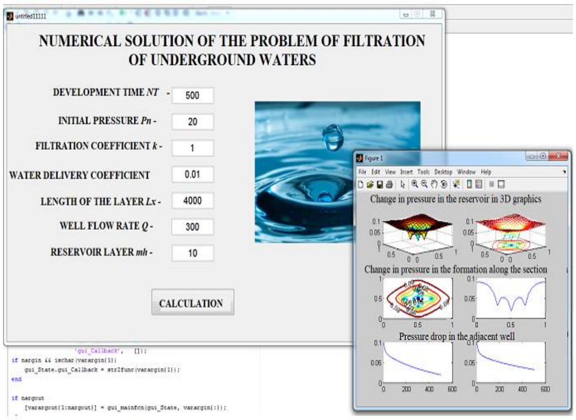

Fig. 1. Program for solving problems of groundwater filtration and visualization of numerical calculations of the computational experiment

The calculation results for the problem of pressure filtration in a single-layer zone are displayed as follows (Fig.1):

- Change in pressure in the formation in a 3D graph in various forms;
- Change in pressure in the formation in a contour graph;
- Change in pressure in graphs in the x-section;
- Drop in pressure in wells in graphs.

The following initial data are used to solve the problem:

n=21- the number of points in the grid area along x and y;

nt=500 - the total time for calculation;

dt=1 - the time step (per day);

pn1=200 - the initial pressure in the reservoir;

k=1 - the filtration coefficient;

μ - the free water loss coefficient;

Lx=4000 - the length of the filtration area along the x and y directions;

Q=300 - the well flow rates;

h=10 - power layer.

4. Conclusion

The software can be used for various similar two-dimensional problems, which mathematical model is described in the form of a differential equation of parabolic type.

References

[1] Davydov L.K., Dmitrieva A.A., Konkina N.G. General hydrology. 2nd ed., revised and supplemented. – Leningrad: Gidrometizdat, 1973. – 464 p.

[2] Abutaliev F.B. [and others]. Application of numerical methods and computers in hydrogeology. Tashkent, “Fan”, 1976.

[3] Belman R., Kalaba R. Quasi-linearization and nonlinear boundary value problems. Mir, Moscow, 1968.

[4] Khuzhayarov B.Kh. Macroscopic simulation of relaxation mass transport in a porous medium // Fluid Dynamics. Vol. 29, No. 5. - 2004.- P. 693-701.

[5] Suzuki A., Horne R.N., Makita H., Niibori Y., Fomin S.A., Chugunov V.A., Hashida T. Development of

fractional derivative-based mass and heat transport model // Proceedings, Thirty-Eighth Workshop on Geothermal Reservoir Engineering Stanford University, Stanford, California, February 11-13. 2013. - SGP−TR−198.

[6] Khuzhayorov B.Kh., Makhmudov J.M. Mathematical models of filtration of heterogeneous liquids in porous media. - Tashkent: Fan, 2014. - 280 p.

[7] Khuzhayorov B., Dzhiyanov T., Khaydarov O. Double-Relaxation Solute Transport in Porous Media // International Journal of Advanced Research in Science, Engineering and Technology. Vol. 5, Issue 1, January 2018. – P. 5094-5100.

[8] Fomin S.A., Chugunov V.A. and Hashida T. Non-Fickian mass transport in fractured porous media // Advances in Water Resources. 34(2).- 2011.- P. 205–214.

[9] Molokovich Yu.M. Nonequilibrium filtration and its application in oil field practice. – M. – Izhevsk: Research Center “Regular and Chaotic Dynamics”; Institute of Computer Research, 2006. – P. 214.

[10] Nikiforov A.I., Sadovnikov R.V. Solution of oil reservoir flooding problems using polymer-dispersed systems on a multiprocessor computing system. Institute of Mechanics and Mechanical Engineering, Kazan Scientific Center of the Russian Academy of Sciences, 2016. Vol. 28, No. 8. – P. 112-126.

[11] Nazirova E.Sh. Mathematical models, numerical methods and software packages for studying the filtration processes of liquids and gases: Diss. Doctor of Engineering Sciences. – Tashkent, 2019. – 227 p.

[12] Samarskii A.A. Introduction to the theory of difference schemes. Moscow: “Nauka”, 1952.

[13] Saidullaev U.Zh. Compilation and numerical analysis of hydrodynamic models of filtration and suspension filtration: Diss. PhD… physical and mathematical sciences. Tashkent, 2019. – 112 p. with appendix.

[14] Makhmudov Zh.M. Improvement and analysis of mathematical models of filtration of heterogeneous liquids in porous media: Diss… Dr. of Phys. and Mathematics. Samarkand, 2019. – 216 p. with appendix

[15] Lykov AV, Mikhailov Yu.A. Theory of heat and mass transfer. M.-L.: Gosenergoizdat, 1963. – 536 p.

[16] Akilov Zh.A., Zhiyanov T.O. Double relaxation transfer of matter in a porous medium // Uzbek journal “Problems of Mechanics”, 2013, No. 2. – P.16-18.

[17] Khuzhayorov B., Dzhiyanov T., Khaydarov O. Double-Relaxation Solute Transport in Porous Media // International Journal of Advanced Research in Science, Engineering and Technology. Vol. 5, Issue 1, January 2018. – P. 5094-5100.

[18] Dzhiyanov T.O. Development and analysis of models of anomalous filtration of heterogeneous liquids in porous media: Diss...PhD in Physics and Mathematics. Tashkent, 2019. – 115 p. with appendix.

[19] Fomin S.A., Chugunov V.A. and Hashida T. Non-Fickian mass transport in fractured porous media // Advances in Water Resources. 34(2).- 2011.- P. 205–214.

[20] Zikiryaev Sh.Kh. Filtration problems of heterogeneous liquids taking into account adsorption and heterogeneity of pore space filling: Abstract of Cand. of Phys. and Mathematics. – Samarkand, 2012. – 42 p.

[21] Molokovich Yu. M. Nonequilibrium filtration and its application in oil field practice. - M. - Izhevsk: Research Center "Regular and Chaotic Dynamics"; Institute of Computer Research, 2006. - P. 214.

[22] Nikiforov A. I., Sadovnikov R. V. Solution of problems of flooding of oil formations using polymer-dispersed systems on a multiprocessor computing system. Institute of Mechanics and Mechanical Engineering of the Kazan Scientific Center of the Russian Academy of Sciences, 2016. Vol. 28, No. 8. - P. 112-126.

[23] Zhuraev Sh. Sh. Evaluation of the effectiveness of local raw materials for reducing the filtration process in soil structures: Abstract of PhD in technical sciences. Tashkent, 2018. - 40 p.

[24] Sulaymonov F. U. Hydrodynamic problems of substance transfer in an aggregated porous medium: Diss. PhD in Phys. and Mathematics. – Samarkand, 2019. – 117 p. with appendix.

[25] Burnashev V. F. Numerical analysis of hydrodynamic models of filtration of multiphase multicomponent fluids in porous media: Diss. Dr. of Phys. and Mathematics. – Samarkand, 2019. - 212 p.

[26] Makhmudov Zh. M. Improvement and analysis of mathematical models of filtration of heterogeneous fluids in porous media: Diss. Dr. of Phys. and Mathematics. Samarkand, 2019. – 216 p. with appendix.

[27] Sylvian J. Pirson. Oil reservoir engineering. – McGraw – Hill Book Co. Inc, New – York – Toronto – London, 1958.

[28] Nigmatulin R. I. Dynamics of multiphase media. – Moscow: Nauka, 1987. Part 1. 464 p., Part 2. 360 p.

[29] Samarskii Vabishchevich. Konstantinov N. M., Petrov N. M., Petrov N. A., Vysotskii L. I. Hydraulics, hydrology, hydrometry / Textbook for universities: in 2 parts. Part II. Special issues - Moscow: Higher. school, 1987 - 431 p.

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