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"QURILISHDA YASHIL IQTISODIYOT, SUV VA ATROF-MUHITNI ASRASH TENDENSIYALARI, EKOLOGIK MUAMMOLAR VA INNOVATSION YECHIMLAR" MAVZUSIDAGI RESPUBLIKA MIQYOSIDAGI ILMIY-AMALIY KONFERENSIYA TASHKILIY QO'MITASI

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Leibniz algebras generated using the image of Euclidean algebras

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Abstract: In this work we have considered about Lie and Leibniz algebras and their representations, particularly we have learned the Euclidean algebra . For a given the Euclidean algebra and the right -module , we can construct a Leibniz algebra. In this paper we have learned Leibniz algebras constructed by Euclidean Lie algebra and some of its modules. We used its modules that arise from matrix realization of .
Keywords: Lie algebras, Leibniz algebras, representation, modul, ideal, matrix.

Yevklid algebrasining tasviri yordamida hosil qilinuvchi Leybnits algebralari

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Annotatsiya: Bu ishda biz Li va Leybnits algebralari va ularning tasvirlarini ko‘rib chiqdik, xususan, Yevklid algebrasini o‘rgandik. Berilgan Yevklid algebrasi va o‘ng -modul uchun biz Leybnits algebrasini qurishimiz mumkin. Ushbu maqolada biz Yevklid Li algebrasi tomonidan tuzilgan Leybnits algebralari va ularning bir modulini o‘rgandik. Biz ning matritsali ifodasidan hosil bo‘lgan modullaridan foydalandik.
Kalit so‘zlar: Li algebralari, Leibniz algebralari, tasvir, modul, ideal, matritsa.

1. Kirish

Li algebralarining umumlashmasi hisoblangan Leybnits algebralari hozirgi kunda Respublikamiz matematiklari va horijiy olimlar tomonidan jadal sur’atda o‘rganilmoqda. Leybnits algebralarini tasniflashda ularni sinflarga ajratib o‘rganish muhim o‘rin tutadi. Ma’lumki, ixtiyoriy Li bo‘lmagan Leybnits algebrasi notrivial idealga ega bo‘lib, bu ideal bo‘yicha faktor algebra Li algebrasi bo‘ladi. Berilgan Li algebralari va ularning tasvirlari orqali Leybnits algebralarini hosil qilish mumkin. Hozirgi kunda Li algebralarining tasvirlari muhim nazariy va amaliy ahamiyatga egadir. $e(4)$ Yevklid algebrasi ham Li algebrasi bo‘lib, uning tasvirlari yordamida Leybnits algebralarini hosil qilish mumkin. Ana shunday tasvirlarni topish va ular orqali hosil qilingan Leybnits algebralarini tasniflash muhim ahamiyatga ega.

J.Q. Adashev va B.A. Omirovlarning ishlarida Yevklid Li algebralarining tasvirlari yordamida hosil qilinuvchi Leybnits algebralari o‘rganilgan.

1-ta’rif. *F maydon ustidagi L vektor fazo, $L\times L\to L$, $(x,y)\mapsto[x,y]$ kabi belgilanib, kommutator deb nomlanuvchi amalga nisbatan quyidagi aksiomalarni qanoatlantirsa, u holda uni F maydon ustidagi Li algebrasi deyiladi:*

(L1) Kommutator amali bichiziqli.

(L2) $[x,x]=0, \forall x\in L.$

(L3) $[x,[y,z]]+[y,[z,x]]+[z,[x,y]]=0\ (x,y,z\in L).$

(*L3*) aksioma Yakobi ayniyati deyiladi. Ta’kidlaymizki, (*L1*) va (*L2*) dan $[x+y,x+y]$ ni hisobga olib, antikommutativlikni olamiz: (*L2’*) $[x,y]=-[y,x]$. (Teskarisi, agar $\mathrm{char}F\neq 2$ bo‘lsa, (*L2*) dan (*L2’*) ni kelib chiqishi ravshan.)

2-ta’rif. *K maydon bo‘lsin. $(L,[-,-])$ algebra K maydon ustidagi Leybnits algebrasi deyiladi, agar ihtiyoriy $x,y,z\in L$ uchun quyidagi Leybnits ayniyati bajarilsa:*

$[x,[y,z]]=[[x,y],z]-[[x,z],y]$.

Ravshanki, ihtiyoriy Li algebrasi Leybnits algebrasidir.

L Leybnits algebrasi uchun, $\varphi: L\to L/I$ tabiiy gomomorfizmni qaraylik. Bu yerda $\bar{L}=L/I$ Li algebrasi bo‘lib, uni L Leybnits algebrasiga mos qo‘yilgan Li algebrasi deyiladi. $I\times\bar{L}\to L$, $(i,\bar{x})\to[i,x]$ akslantirish o‘ng $\bar{L}$- modul bo‘ladi.

$Q(L)=\bar{L}\oplus I$ bo‘lsin, u holda $(-,-)$amal $Q(L)$ da Leybnits algebrasini aniqlaydi, bunda

$$(\bar{x},\bar{y})=\overline{[x,y]},\ (\bar{x},i)=0,\qquad (i,\bar{x})=[i,x],$$
$$(i,j)=0,\qquad x,y\in L,\ i,j\in I.$$

Shunday qilib, berilgan G Li algebrasi va o‘ng G -modul M uchun, yuqoridagidek Leybnits algebrasini qurishimiz mumkin. [4].

$e(4)$ Yevklid algebrasi $\{E_{i,j},H_k|i<j\}$ bazisga ega bo‘lib, uning ko‘paytirish jadvali quyidagicha**:**

$$[E_{i,j},E_{j,k}]=E_{i,k},[E_{i,j},H_j]=H_i,$$
$$[E_{i,j},H_i]=-H_j,$$

Bu yerda $E_{i,j}=-E_{j,i}$. [4]

$e(4)$ Yevklid algebrasini matritsaviy formada quyidagicha yozishimiz mumkin:

$$\begin{pmatrix} & & & & x_1\\ & A & & & x_2\\ & & & & x_3\\ & & & & x_4\\ 0&0&0&0&0\end{pmatrix},$$

bunda A kososimmetrik matritsa. Bazis elementlari quyidagilarga teng:

$$E_{i,j}=e_{i,j}-e_{j,i}, 1\leq i\neq j\leq 4,$$

$$H_k = e_{k,5}, 1 \le k \le 4,$$

bunda $e_{i,j}$ matritsalar i -satr va j -ustunning kesishishida bir bo'lib, qolgan elementlari nol bo'lgan matritsa.

$\frac{L}{I}$ faktor Li algebrasi and va uning tabiiy gomomorfizmidagiga proobrazini farqlash uchun biz faktor algebraning ustiga chiziqcha qo'yib belgilaymiz.

Endi biz $\frac{L}{I \cong \bar{e}(4)}$ shartni qanoatlantiruvchi L Leybnits algebralarini tasvirlaymiz (bu yerda $\bar{e}(4)$ algebra $e(4)$ algebrani anglatadi) va I ideal 5-o'lchamli o'ng $\bar{e}(4)$ -modul bo'lsin, va uning bazisi $\{X_1, X_2, X_3, X_4. X_5\}$ ga teng bo'lsin. Modulni quyidacha aniqlaymiz:

$$[X_i, \bar{E}_{i,j}] = X_j, \quad 1 \le i, j \le 4,$$

$$[X_i, \bar{H}_i] = X_5, \quad 1 \le i \le 4. \qquad (1)$$

Ta'kidlash joizki, $\bar{e}(4) \cong so_3$, bunda so_3 sodda ortogonal Li algebrasi bo'lib, uning bazisi $\bar{E}_{i,j}, i < j$.

Shunday qilib biz $L \cong so_3 + I$ algebraga ega bo'lamiz, uning bazisi 15-o'lchamli bo'lib, quyidagiga teng: $\{E_{i,j}, i < j, H_k, X_1, X_2, X_3, X_4, X_5\}$, bunda $E_{i,j}, H_k$ elementlar mos faktor algebra $\frac{L}{I}$ ning proobrazlari. Levi teoremasiga asosan so_3 ni L ning qism algebrasi deb hisoblashimiz mumkin ya'ni, , $[E_{i,j}, E_{j,k}] = E_{i,k}$.

Teorema. *L $\frac{L}{I \cong \bar{e}(4)}$ bo'ladigan Leybnits algebrasi bo'lsin va and I ideal (1) kabi aniqlangan o'ng $\bar{e}(4)$ -modul bo'lsin. U holda* $[e(4), e(4)] = e(4)$.

Isbot. Biz quyidagilarni tuzamiz:

$$[E_{i,j}, H_i] = -H_j + \sum_{t=1}^{5} A_{i,j,i}^t X_t,$$

$$[H_i, E_{i,j}] = H_j + \sum_{t=1}^{5} B_{i,i,j}^t X_t,$$

$$[E_{i,j}, H_j] = H_i + \sum_{t=1}^{5} A_{i,j,j}^t X_t,$$

$$[H_j, E_{i,j}] = -H_i + \sum_{t=1}^{5} B_{j,i,j}^t X_t,$$

$$[E_{i,j}, H_k] = \sum_{t=1}^{5} A_{i,j,k}^t X_t, k \notin \{i,j\},$$

$$[H_k, E_{i,j}] = \sum_{t=1}^{5} B_{k,i,j}^t X_t, k \notin \{i,j\},$$

$$[H_i, H_j] = \sum_{t=1}^{5} C_{i,j}^t X_t.$$

Quyidagi almashtirishlarni olamiz:

$$H_1' = H_1 + \sum_{t=1}^{5} A_{1,2,2}^t X_t,$$

$$H_j' = H_j - \sum_{t=1}^{5} A_{1,j,1}^t X_t, 2 \le j \le 4,$$

Biz

$$[E_{1,2}, H_2] = H_1, \quad [E_{1,j}, H_1] = -H_j, 2 \le j \le 4,$$

deb hisoblashimiz mumkin.

$2 \le i \ne j \le 4$ uchun Leybnits algebrasini tekshiramiz: $[E_{j,i}, [E_{1,i}, H_1]] = [[E_{j,i}, E_{1,i}], H_1] - [[E_{j,i}, H_1], E_{1,i}] =$

$$= -H_j - A_{j,i,1}^1 X_i + A_{j,i,1}^i X_1.$$

Ikkinchi tomondan quyidagiga ega bo'lamiz:

$$[E_{j,i}, [E_{1,i}, H_1]] = -[E_{j,i}, H_i] = [E_{i,j}, H_i].$$

Natijada,

$$[E_{i,j}, H_i] = -H_j + A_{j,i,1}^i X_1 - A_{j,i,1}^1 X_i,$$
$$2 \le i \ne j \le 4. \qquad (2)$$

Shunga o'xshash, quyidagiga ega bo'lamiz:

$$[E_{k,l}, H_i] = -A_{k,l,1}^i X_1 + A_{k,l,1}^1 X_i,$$
$$2 \le i, k, l \le 4, i \notin \{k, l\}. \qquad (3)$$

(2) ga ko'ra va $A_{i,j,1}^1 = -A_{j,i,1}^1$ tengliklarni quyidagi uchliklar uchun tekshirilgan Leybnits ayniyatlariga qo'yamiz

$$\{E_{i,j}, H_i, E_{i,j}\}, \ 2 \le i \ne j \le 4,$$

$$\{E_{i,j}, H_i, E_{k,l}\}, \ 1 \le i \ne j \le 4,$$

$$2 \le k \ne l \le 4, \ \{i,j\} \cap \{k,l\} = \{0\},$$

biz bundan quyidagiga ega bo'lamiz

$$\begin{cases} [H_j, E_{i,j}] = -H_i + A_{i,j,1}^j X_1, \\ \ 2 \le i \ne j \le 4, \\ [H_j, E_{k,l}] = 0, \\ 1 \le j \le 4, \ 2 \le k \ne l \le 4, \ j \notin \{k,l\}. \end{cases} \qquad (4)$$

(2)-(3) lardan foydalanib$\{E_{i,j}, H_k, E_{k,l}\}$ uchliklar uchun Leybnits ayniyatidan foydalanib quyidagini hosil qilamiz:

$$A_{i,j,1}^1 = 0, \ 2 \le i \ne j \le 4 \Rightarrow$$

$$[E_{k,l}, H_i] = -A_{k,l,1}^i X_1, \ 2 \le i,k,l \le 4, \ i \notin \{k,l\}.$$

Xuddi shu kabi, (3)-(4) dan va $\{E_{1,2}. E_{i,2}, H_i\}$, $\{E_{i,j}, E_{1,2}, H_2\}$, $3 \le i, j \le 4$ uchliklar uchun Leybnits ayniyatida quyidagini olamiz:

$$[E_{i,1}, H_i] = -H_1 + A_{1,2,i}^1 X_2 - A_{1,2,i}^2 X_i,$$

$$[E_{i,j}, H_1] = A_{i,j,1}^2 X_2, \ 3 \le i, j \le 4.$$

Demak

$$0 = [E_{i,j}, [H_1, H_2]] =$$
$$[[E_{i,j}, H_1], H_2] - [[E_{i,j}, H_2], H_1] = 2A_{i,j,1}^2 X_2$$

bundan

$$A_{i,j,1}^k = 0, \ 1 \le k \le 4,$$
$$3 \le i, j \le 4 \Rightarrow [E_{i,j}, H_1] = 0.$$

Shunday qilib, quyidagini olamiz:

$$[E_{i,j}, H_i] = -H_j, \quad 3 \le i \ne j \le 4,$$

$$[H_j, E_{i,j}] = -H_i, \quad 3 \le i \ne j \le 4,$$

$$[H_j, E_{k,l}] = 0, 1 \le j \le 4,$$
$$2 \le k \ne l \le 4, \ j \notin \{k,l\}.$$

$3 \le i, j, k \le 4$ uchun Leybnits ayniyatini tekshiramiz:

$$[E_{i,1}, [E_{j,k}, H_j]] =$$
$$= [[E_{i,1}, E_{j,k}], H_j] - [[E_{i,1}, H_j], E_{j,k}]$$
$$= A_{i,1,j}^j X_k - A_{i,1,j}^k X_j.$$

Ikkinchi tomondan, quyidagiga ega bo'lamiz:

$$[E_{i,1}, [E_{j,k}, H_j]] = -[E_{i,1}, H_k].$$

Shunday qilib, quyidagini xulosa qilamiz:

(5) ning o'ng tomonida erkin parametr j mavjud va (5) ning chap tomonida yo'q bo'lganligi uchun $[E_{i,1}, H_k] = 0$ bo'lishi kelib chiqadi.

Quyidagi uchliklar uchun Leybnits ayniyatini qo‘llanib, quyidagilarga ega bo‘lamiz:

$\{E_{j,i}, H_j, E_{1,k}\},\ 3 \le i, j, k \le 4$

$\Rightarrow [H_j, E_{1,i}] = 0, \qquad 3 \le i, j \le 4,$

$\{E_{2,j}, H_2, E_{2,j}\},\ 3 \le j \le 4$

$\Rightarrow [H_j, E_{2,j}] = -H_2, \qquad 3 \le j \le 4,$

$\{E_{i,1}, H_i, E_{i,j}\},\ 3 \le i \ne j \le 4$

$\Rightarrow A^2_{1,2,i} = 0, \qquad 3 \le i \le 4,$

$\{E_{2,j}, H_2, E_{1,k}\},\ 3 \le j, k \le 4$

$\Rightarrow A^2_{j,2,1} = 0, \qquad 3 \le j \le 4,$

$\{E_{i,j}, E_{j,2}, H_1\},\ 3 \le i \ne j \le 4$

$\Rightarrow [E_{i,2}, H_1] = 0, \qquad 3 \le i \le 4,$

$\{E_{i,j}, E_{j,1}, H_2\},\ 3 \le i \ne j \le 4$

$\Rightarrow [E_{i,1}, H_2] = 0, \qquad 3 \le i \le 4,$

$\{E_{i,1}, E_{1,2}, H_1\},\ 3 \le i \le 4$

$\Rightarrow [H_i, E_{1,2}] = 0, \qquad 3 \le i \le 4,$

$\{E_{i,1}, H_i, E_{2,j}\},\ 3 \le i \ne j \le 4$

$\Rightarrow [H_1, E_{2,j}] = A^i_{1,2,j} X_j,\ 3 \le i \ne j \le 4.$

Quyidagini qaraylik

$\{E_{1,2}, H_i, E_{i,j}\},\ 3 \le i \ne j \le 4 \Rightarrow$

$$[E_{1,2}, H_j] = A^i_{1,2,i} X_j - A^j_{1,2,i} X_i,\ 3 \le i \ne j \le 4. \quad (6)$$

(6) dan foydalanib

$\{E_{1,2}, H_j, E_{i,l}\},\ \ \{E_{1,3}, E_{3,2}, H_i\},\ 3 \le i, j \le 4,$

uchliklar uchun quyidagiga ega bo‘lamiz:

$A^i_{1,2,i} = A^j_{1,2,i} = 0,\ 3 \le i, j \le 4.$

Quyidagi uchliklar uchun Leybnits ayniyatini qo‘llaymiz

$\{E_{i,2}, H_j, E_{2,j}\},\ \ \{E_{i,1}, H_i, E_{1,j}\},$

$\{E_{k,i}, H_k, E_{i,2}\},\ \ \{E_{k,i}, H_k, E_{i,1}\}$

va bundan $[H_i, E_{i,j}] = H_j, 1 \le i \ne j \le 4.$

Endi qolgan ko‘paytmalarning nolligini ko‘rsatamiz, ya’ni $[H_i, H_j] = 0.$

Leybnits ayniyatidan quyidagiga ega bo‘lamiz:

$\{E_{i,k}, H_i, H_j\},\ 1 \le i \ne j \le 4$

$\Rightarrow [H_i, H_j] = 0, \qquad 1 \le i \ne j \le 4,$

$\{H_i, E_{i,j}, H_j\},\ 1 \le i \ne j \le 4$

$\Rightarrow [H_1, H_1] = [H_i, H_i], \qquad 1 \le i \le 4,$

$\{E_{1,i}, H_1, H_i\},\ 1 \le i \le 4$

$\Rightarrow [H_1, H_1] = -[H_i, H_i], \qquad 1 \le i \le 4.$

Shunday qilib, biz $[H_i, H_j] = 0,\ \ 1 \le i,j \le 4$, bo‘lishini hosil qilamiz, bu esa teoremaning isbotini tugallaydi.

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