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Modern approach to mathematical modeling of thermal processes in the axle box of rolling stock

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Abstract: This study develops a mathematical model for thermal processes in railway axle box assemblies to enhance operational safety and reliability. The model applies fundamental heat transfer principles—Fourier's law, heat conduction, and energy balance accounting for internal heat generation, convection, and radiation. Using a homogeneous temperature distribution assumption, the problem is simplified to a first-order differential equation. Numerical implementation via the Euler method in MATLAB yielded quantitative temperature dynamics, stationary values, and system settling time. Comparative analysis of convective and radiative losses was performed with graphical visualization. Results confirm the model's adequacy to real operating conditions. The work provides a rigorously formalized approach suitable for developing non-contact monitoring systems and integration into intelligent diagnostic complexes, representing significant practical and scientific novelty.

Keywords: Mathematical model, thermal processes, axle box assembly, railway rolling stock, Fourier's law, heat conduction, heat balance, convection and radiation, numerical modeling, Euler method

1. Introduction

The operational safety of railway rolling stock is largely determined by the technical condition of the axle box assemblies, which are among the most heavily loaded components of the wheelset. Overheating of the axle box bearing leads to increased wear, reduced reliability of the unit, and in some cases, can cause emergency situations involving derailment. According to international practice, approximately 8–10% of failures on mainline railways are associated with abnormal heating of axle box assemblies, which confirms the relevance of the tasks of diagnosing and monitoring their thermal condition.

Modern systems for the technical diagnostics of rolling stock include various approaches: the use of contact sensors, infrared temperature measuring devices, as well as non-contact remote monitoring systems. However, most existing methods are empirical in nature and do not allow for sufficiently accurate prediction of the axle box heating dynamics under real operational conditions. In this regard, the development of a rigorous mathematical model that accounts for the physical processes of heat transfer and allows not only for describing the thermal behavior of the unit but also for predicting its state over time is of particular importance.

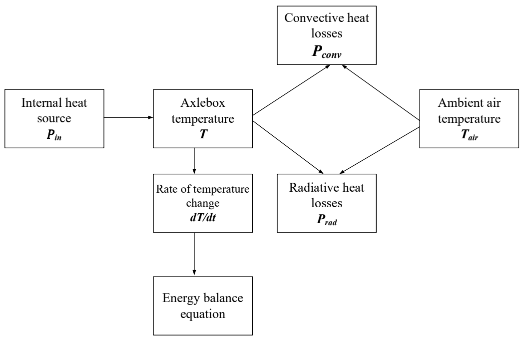

Fig. 1. Structure of the Axle Box Thermal Model

The aim of this study is to develop a modern mathematical model of the thermal processes in the axle box assembly of railway rolling stock. To achieve this aim, the following tasks are addressed:

- Formalization of heat transfer processes based on Fourier's law and the heat conduction equation;
- Development of a simplified heat balance model for the axle box assembly;
- Implementation of a numerical solution to the problem in the MATLAB environment using the Euler method;
- Analysis of the obtained results and assessment of the model's applicability for the tasks of diagnostics and monitoring of the technical condition.

Thus, the relevance of the research is defined by the need to enhance traffic safety through the application of rigorous mathematical models in axle box monitoring systems, and the scientific novelty lies in the development of a new approach to describing their thermal processes with the potential for practical implementation.

2. Research methodology

Theoretical background of thermal processes

To construct a rigorous model of the thermal behavior of the axlebox, it is necessary to rely on the fundamental laws of heat transfer. The basis consists of processes of conduction, convection, and radiation, described by the classical equations of mathematical physics.

$$\vec{q} = -k\nabla T, \quad (1)$$

where k is the thermal conductivity and ∇T is the temperature gradient.

The general transient heat conduction equation is:

$$\rho c\frac{\partial T}{\partial t} = \nabla(k\nabla T) + F(x,t), \quad (2)$$

where ρ is density, c is specific heat capacity, and F(x,t) represents internal heat sources.

For a homogeneous material with constant properties the equation takes the form:

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$$\frac{\partial T}{\partial t} = a^2 \frac{\partial^2 T}{\partial x^2} + \frac{F(x,t)}{\rho c}, \quad (3)$$

where $a^2 = \frac{k}{\rho c}$ is the thermal diffusivity.

Simplification of the model for the axlebox

Despite the complexity of real processes, for engineering analysis it is reasonable to introduce certain simplifying assumptions. Due to the high thermal conductivity of the metal, the axlebox temperature can be considered spatially uniform, which significantly reduces the dimensionality of the problem.

$$C\frac{dT}{dt} = P_{\text{in}} - P_{\text{conv}}(T) - P_{\text{rad}}(T), \quad (4)$$

where C is the heat capacity of the axlebox, P_{in} is the power of heat generation, and P_{conv}, P_{rad} are convective and radiative heat losses, respectively.

Heat losses

The main mechanisms of heat dissipation are convection into the ambient environment and thermal radiation. In this model, both processes are formalized analytically and included in the energy balance of the axlebox.

Convection is described by Newton's law of cooling:

$$P_{\text{conv}}(T) = hA(T - T_{\text{air}}), \quad (5)$$

where hhh is the convective heat-transfer coefficient, A is the surface area, and T_{air} is the ambient temperature.

Radiation is expressed by the Stefan–Boltzmann law:

$$P_{\text{rad}}(T) = \varepsilon\sigma A[(T + 273.15)^4 - (T_{\text{air}} + 273.15)^4], \quad (6)$$

where ε is the surface emissivity, and $\sigma = 5.67 \cdot 10^{-8}\,\text{W}/(\text{m}^2 \cdot \text{K}^4)$.

Governing differential equation

Combining internal heat generation and heat losses yields a nonlinear first-order differential equation. It describes the transient thermal dynamics of the axlebox and constitutes the core of the developed model.

$$\frac{dT}{dt} = \frac{1}{C}(P_{\text{in}} - hA(T - T_{\text{air}}) - \varepsilon\sigma A[(T + 273.15)^4 - (T_{\text{air}} + 273.15)^4]). \quad (7)$$

Steady-state regime

Analysis of the steady-state regime allows determining the equilibrium temperature of the axlebox under given operating conditions. This has practical importance for estimating the permissible operating range and predicting overheating.

$$P_{\text{in}} = hA(T_* - T_{\text{air}}) + \varepsilon\sigma A[(T_* + 273.15)^4 - (T_{\text{air}} + 273.15)^4] \quad (8)$$

Criterion for reaching steady-state

Besides the steady-state value of the temperature, it is essential to estimate the time required for the system to reach equilibrium. For this purpose, the 95% criterion of the transient temperature difference is employed:

$$T(t) \geq T_0 + 0.95\,(T_* - T_0), \quad (9)$$

where T_0 is the initial axlebox temperature.

Numerical implementation

Since analytical solutions of equation (7) are difficult due to its nonlinear character, a numerical approach is applied. The Euler method is used to simulate the heating dynamics, providing step-by-step approximation of the temperature evolution over time.

$$T_{i+1} = T_i + \left(\frac{dT}{dt}|_{T_i}\right)\Delta t, \quad (10)$$

where Δt is the integration step.

The implementation was carried out in MATLAB, which enabled the construction of temperature trajectories and quantitative assessment of transient processes.

3. Results and Discussion

3.1. Temperature dynamics

The numerical solution of equation (7) was obtained using the explicit Euler method with a time step of Δt=1 s. The simulation results demonstrate a monotonic increase in axlebox temperature from the initial value T_0 to the steady-state regime T∗.

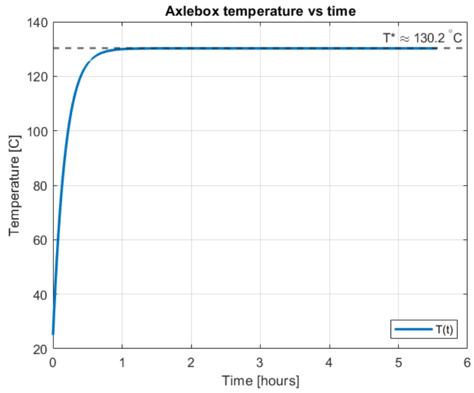

Fig. 2. Temperature evolution over time

Figure 2 presents the temporal dependence T(t), which shows a smooth transition from the initial to the stationary temperature. The heating process exhibits an asymptotic behavior typical of first-order thermal systems.

3.2. Steady-state temperature

The steady-state value T∗ was determined from equation (8). The analysis revealed that the equilibrium temperature depends linearly on the convective parameters (h,Ah,) and nonlinearly on the radiative characteristics (ε,σ).

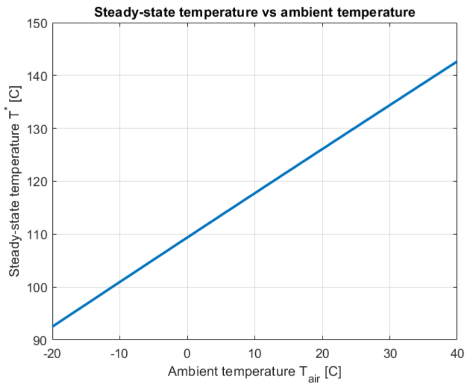

Fig. 3. Dependence of the steady-state temperature on ambient temperature

Figure 3 illustrates the dependence of the steady-state axlebox temperature on the ambient temperature. It was established that increasing external temperature significantly raises the equilibrium value, thereby increasing the risk of axlebox overheating during operation.

3.3. Time to steady-state

To estimate the transient behavior, criterion (9) was applied, corresponding to the achievement of 95% of the steady-state temperature. The calculations showed that the time required to reach steady-state is approximately

$t_{95} \approx 5000 - 6000$ s,

which corresponds to about 1.5–2 hours of operation.

Figure 4 depicts the process of reaching the 0.95 T_* level, with the moment of attaining the steady-state condition clearly marked.

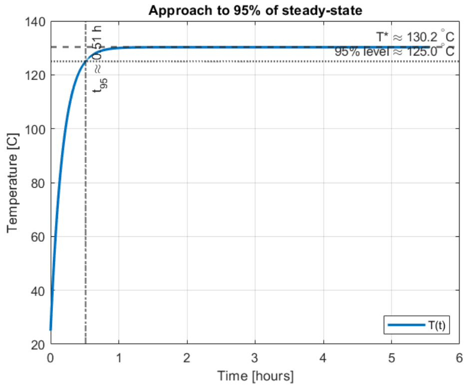

Fig. 4. Transient behavior and determination of the time t_{95}

3.4. Analysis of heat losses

A comparative analysis of convective and radiative heat losses was performed. It was found that at temperatures below 80°C convection is the dominant mechanism, whereas at temperatures above 120°C radiation becomes the primary heat dissipation mechanism. This effect is explained by the fourth-power dependence of radiative heat losses on absolute temperature, as described by the Stefan–Boltzmann law.

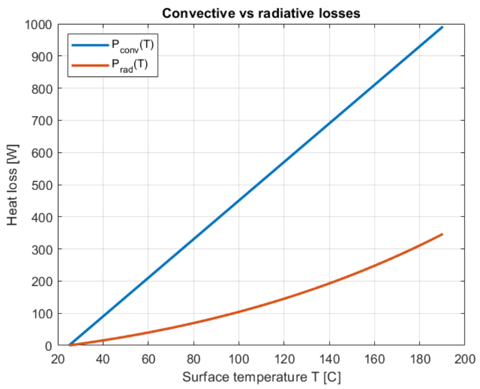

Fig. 5. Distribution of heat losses between convection and radiation

Figure 5 presents the relative contributions of convection and radiation to the total heat balance. It is evident that radiation dominates at elevated surface temperatures.

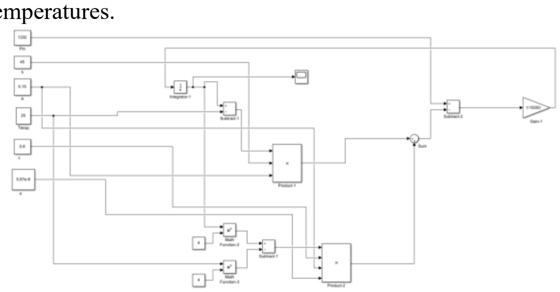

Fig. 6. Block diagram of the thermal model of the axle box in MATLAB

In addition, to verify the developed mathematical model, a block diagram was constructed in **MATLAB/Simulink**. **Figure 7** shows the structure of the Simulink model, while **Figure 6** presents the oscillogram of the axlebox temperature obtained during simulation.

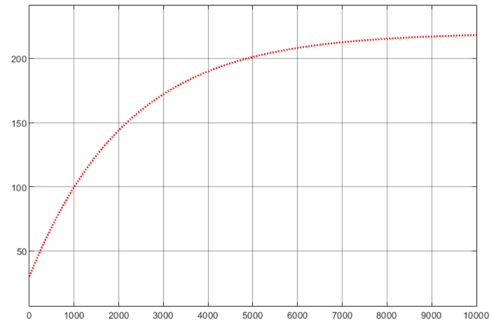

Fig. 7. Oscillogram of the thermal model on the MATLAB

The set of results shown in Figures 1–6 confirms the validity of the proposed model and its ability to adequately describe the actual thermal processes in the axlebox. The findings demonstrate the applicability of the model for predictive diagnostics of axlebox condition and its potential integration into modern intelligent monitoring systems of railway rolling stock.

Discussion. The results of the performed simulations confirm the correctness and adequacy of the developed mathematical model of thermal processes in the axlebox of railway rolling stock. The heating dynamics presented in Figures 2–4 demonstrate behavior characteristic of first-order systems: the temperature increases monotonically and asymptotically approaches its steady-state value.

A comparative analysis of heat losses (Figure 5) revealed a regular transition from the predominance of convective heat transfer at moderate temperatures to the dominance of radiative heat exchange at temperatures above 120°C. This phenomenon is fully consistent with the fundamental laws of heat transfer and emphasizes the necessity of accounting for radiative losses when modeling the thermal regime of the axlebox.

Additional verification of the model, performed in MATLAB/Simulink (Figures 6–7), demonstrated a high degree of agreement between the numerical results obtained by analytical and simulation methods. This confirms the validity of the proposed mathematical formulation and its applicability as a reliable tool for engineering calculations and predictive analysis.

The developed model offers several significant advantages compared to traditional empirical approaches:

- it is based on rigorous physical principles (Fourier's law, the heat conduction equation, Stefan–Boltzmann law);
- it is computationally simple due to the use of the uniform-temperature hypothesis;
- it allows both analytical evaluations and numerical experiments to be conducted in modern engineering software environments.

At the same time, the model has certain limitations arising from the assumptions made. In particular, the hypothesis of uniform temperature distribution is valid only for small Biot numbers ($Bi<0.1$). Under conditions of high thermal loads or complex axlebox geometry, considerable temperature gradients may arise, which are not accounted for in the present formulation. Furthermore, the model does not explicitly consider the influence of lubricants, dynamic

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mechanical loads, or variations in the convective heat transfer coefficient associated with changes in train speed and external conditions.

Despite these limitations, the proposed mathematical model constitutes an effective tool for engineering analysis and predictive diagnostics of axlebox condition. Its application is advisable within intelligent monitoring and predictive maintenance systems, thereby contributing to an increased level of operational safety and reliability of railway rolling stock.

4. Conclusion

In this study, a mathematical model of thermal processes in the axlebox of railway rolling stock was developed and verified. The model is based on the fundamental laws of heat transfer (Fourier's law, the heat conduction equation, and the Stefan–Boltzmann law) and incorporates both convective and radiative mechanisms of heat exchange.

The numerical analysis made it possible to determine the temperature dynamics, calculate steady-state values, and estimate the time required to reach thermal equilibrium. A comparative study revealed the transition from convection-dominated heat transfer at moderate temperatures to radiation-dominated heat exchange at temperatures above 120°C.

Verification in MATLAB/Simulink confirmed the validity of the proposed model and demonstrated its applicability for engineering calculations and diagnostic systems. The scientific novelty of the work lies in the rigorous formalization of axlebox thermal processes into a universal mathematical model, which combines analytical precision with computational simplicity.

The practical significance of the research consists in the possibility of applying the model for predictive diagnostics of axlebox condition and its integration into intelligent monitoring systems. This creates a foundation for improving operational safety and enhancing the reliability of railway rolling stock.

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